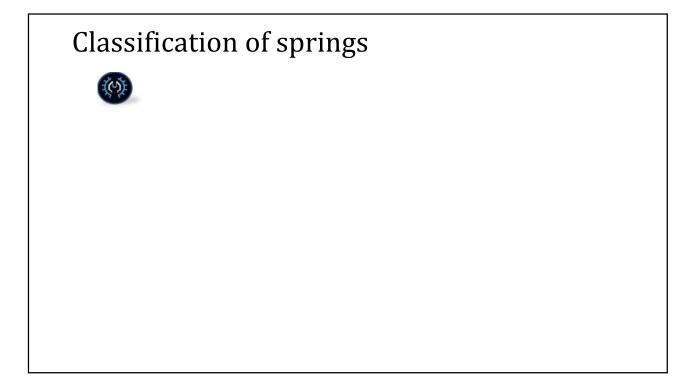
# Design of Machine Elements 2095604

#### **Design of Springs**

- Syllabus:
  - Classification and spring materials
  - Design of helical springs
    - Compression/extension
  - Helical Torsion springs
  - Leaf springs
  - Surge in springs
  - Nipping
  - Shot peening.

## Springs-Introduction

- Spring is defined as elastic machine element, which deflects under the action of load and returns to original shape when load is removed.
- The important functions of springs are:
  - Absorb shock and vibrations
  - Store energy
  - Measure force
  - To apply force and control motion.

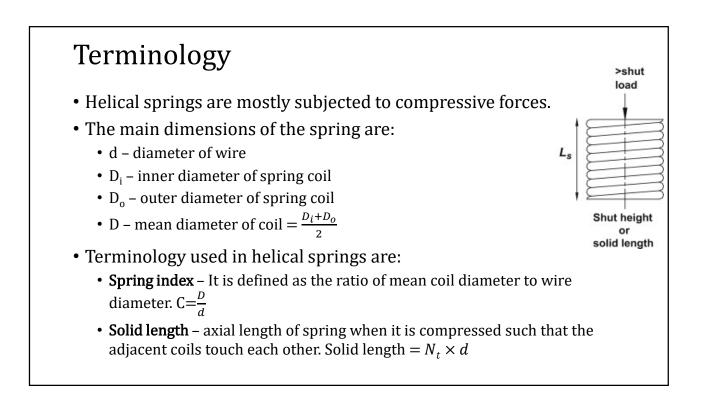


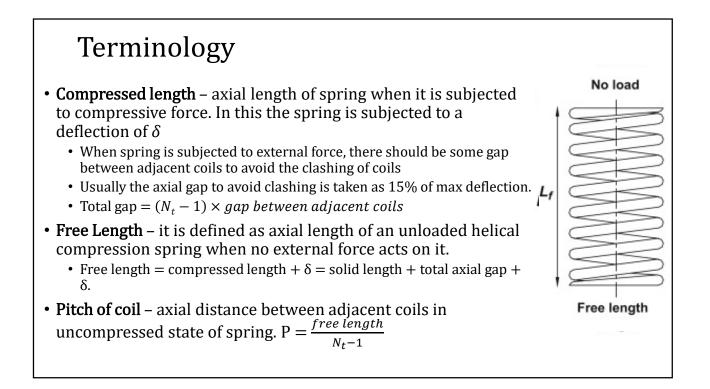
## Helical springs

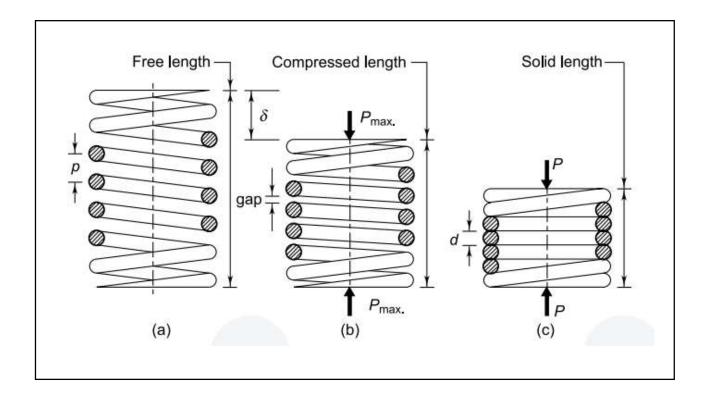
- Helical springs are usually made of wire, circular in cross section which is bent in form of helix.
- These are classified into two types:
  - Compression springs and Extension springs
  - Close coiled springs and open coiled springs.
- Advantages:
  - Easy manufacture
  - Cheap in price
  - High reliability
  - Deflection is linearly proportional to force.

## **Helical Torsion Spring**

- A helical spring which is subjected to torsional load about the axis of coil is called helical torsion spring.
- This spring is used to transmit torque to particular component in a machine.
- These types of springs are used in door hinges, locks etc.
- The helical torsion springs are subjected to bending stresses.
- Note: spring is subjected to torsional moment, but the wire of spring is not subjected to torsional shear stress. It is subjected to bending stresses







#### Active and Inactive coils

- Active coils in spring contribute to spring action, supporting the external force and deflects under the action of it.
- Whereas the coils of spring which doesnot contribute to spring action are called in Inactive coils.
- No of inactive coils = Total number of coils No of active coils.

## Spring materials

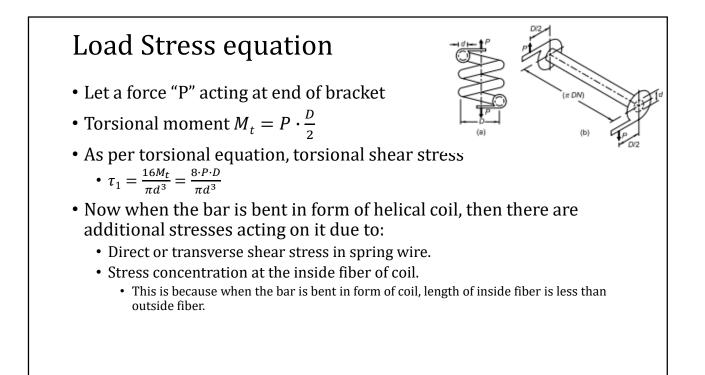
- The selection of spring materials depends on the following factors:
  - Load acting
  - Range of stress
  - Expected fatigue life
  - Environmental conditions.
- There are 4 basic variety of steel wires majorly used in most applications:
  - Cold drawn steel wires
  - Oil hardened and tempered spring steel wires (un alloyed)
  - Oil hardened and tempered spring steel wires (alloyed)
  - Stainless steel spring wires.

## Design of helical springs

- The objectives for design of helical springs are:
  - It should possess sufficient strength to withstand external load
  - It should have required load deflection characteristics
  - It should not buckle under external load.
- Design parameters are:
  - Wire diameter
  - Mean coil diameter
  - Number of active turns
- There are 2 basic equations for design of springs
  - Load stress equation
  - Load deflection equation.

#### **Design of Helical Springs**

- Situation:
- Consider a helical springs which is made of wire of circular crosssection. Let D, d are the mean diameter of coil and wire.
- Let N be number of active coils in spring and is subjected to axial force "P".
- When spring is uncoiled and straightened, it takes shape of bar, therefore it is assumed that a bar is considered for analysis assuming it as equivalent to helical spring.
- Equivalent dimensions are:
  - Diameter of bar = diameter of wire = d
  - Equivalent length of bar =  $N \cdot \pi \cdot D$
  - The bar is fitted with a bracket at each end, the length of this bracket equal to mean radius of spring  $=\frac{D}{2}$



## Design of Helical Springs – load stress eq<sup>n</sup>

- The above equation do not take these effects into consideration there by it requires modification.
- Let
  - $k_s =$  factor to account for direct shear
  - $k_c =$  factor to account stress concentration.
- The combined effect of these 2 factors is given by  $k = k_s \times k_c$
- Hence the equation is modified as  $\tau = k \left[ \frac{8 \cdot P \cdot D}{\pi d^3} \right]$ 
  - k stress factor or wahl factor

• 
$$k = \frac{4C-1}{4C-4} + \frac{0.615}{C}$$

• Note: While designing spring usually the factor of safety is taken as 1.5 or less.

#### Load deflection equation

• The angle of twist ( $\theta$ ) for the equivalent bar is  $\theta = \frac{M \cdot L}{I \cdot G}$ 

• 
$$\theta = \frac{\left(P \cdot \frac{D}{2}\right) \cdot (\pi DN)}{\frac{\pi \cdot d^4}{32} \cdot G} = \frac{16PD^2N}{Gd^4}$$

• Axial deflection of spring  $\delta = \theta \times length \ of \ bracket = \theta \times \frac{D}{2}$ 

• Therefore deflection of spring  $\delta = \frac{8PND^3}{Gd^4}$ 

- It is required to design a helical compression spring subjected to a maximum force of 1250 N. The deflection of the spring corresponding to the maximum force should be approximately 30 mm. The spring index can be taken as 6. The spring is made of patented and cold-drawn steel wire. The ultimate tensile strength and modulus of rigidity of the spring material are 1090 and 81370 N/mm<sup>2</sup> respectively. The permissible shear stress for the spring wire should be taken as 50% of the ultimate tensile strength. Design the spring and calculate:
- (i) Wire diameter;
- (ii) Mean coil diameter;
- (iii) Number of active coils;
- (iv) Total number of coils;
- (v) Pitch of the coil

- Data given:
  - Load, P = 1250 N
  - Deflection,  $\delta = 30 \ mm$
  - Spring index, C = 6
  - Ultimate tensile strength,  $S_{ut} = 1090 \frac{N}{mm^2}$
  - Modulus of rigidity,  $G = 81370 \frac{N}{mm^2}$
  - Shear stress,  $\tau = 0.5S_{\mu t}$
- Solution:

• Wire diameter (d)

- formulae:  $\tau = k \left[ \frac{8 \cdot P \cdot D}{\pi d^3} \right] = k \left[ \frac{8 \cdot P \cdot C}{\pi d^2} \right]$  Where  $K = \frac{4C 1}{4C 4} + \frac{0.615}{C} = \frac{24 1}{24 4} + \frac{0.615}{6} = 1.2525$  Shear stress,  $\tau = 0.5S_{ut} = 0.5 \times 1090 = 545 \frac{N}{mm^2}$   $d = \sqrt{k \times \left( \frac{8 \times P \times C}{\tau \times \pi} \right)} = \sqrt{1.2525 \times \left( \frac{8 \times 1250 \times 6}{545 \times \pi} \right)} = 6.63 \text{ mm} \approx 7 \text{ mm}$

- Data given:
  - Load, P = 1250 N
  - Deflection,  $\delta = 30 \, mm$
  - Spring index, C = 6
  - Ultimate tensile strength,  $S_{ut} = 1090 \frac{N}{mm^2}$
  - Modulus of rigidity,  $G = 81370 \frac{N}{mm^2}$
  - Shear stress,  $\tau = 0.5S_{ut}$
- Solution:
  - Coil diameter (D)
    - Formulae: Spring Index =  $\frac{D}{d} = 6$
    - Hence coil diameter = D = 42 mm
  - Number of active coils:
    - deflection of spring  $\delta = \frac{8PND^3}{Gd^4} = \frac{8PNC^3}{Gd} = 30 \ mm$  Hence,  $N = \frac{\delta Gd}{8PC^3} = \frac{30 \times 81370 \times 7}{8 \times 1250 \times 6^3} = 7.91 \cong 8$

- Data given:
  - Load, P = 1250 N
  - Deflection,  $\delta = 30 \, mm$
  - Spring index, C = 6
  - Ultimate tensile strength,  $S_{ut} = 1090 \frac{N}{mm^2}$
  - Modulus of rigidity,  $G = 81370 \frac{N}{mm^2}$
  - Shear stress,  $\tau = 0.5S_{ut}$
- Solution:
  - Total number of coils
    - Number of inactive coils are 2
    - So the total number of coils = 8 + 2 = 10
  - Pitch of the coil
    - Formulae: pitch =  $\frac{Free \ length}{(total \ number \ of \ coils \ -1)} = \frac{Solid \ length + deflection}{total \ number \ of \ coils \ -1}$
    - Solid length = Total number of coils  $\times$  wire diameter =  $10 \times 7 = 70 \text{ mm}$
    - Pitch  $=\frac{70+30}{10-1} = 11.15 \ mm \cong 12 \ mm$

- A helical compression spring, made of circular wire, is subjected to an axial force, which varies from 2.5 kN to 3.5 kN. Over this range of force, the deflection of the spring should be approximately 5 mm. The spring index can be taken as 5. The spring has square and ground ends. The spring is made of patented and cold-drawn steel wire with ultimate tensile strength of 1050 N/mm<sup>2</sup> and modulus of rigidity of 81370 N/mm<sup>2</sup>. The permissible shear stress for the spring wire should be taken as 50% of the ultimate tensile strength. Design the spring and calculate the following parameters
  - Wire diameter
  - Mean coil diameter

#### Data Given:

- Number of active coils
- Solid length of spring
- Free length of spring
- Spring rate
- Actual spring rate
- 1. Axial force,  $P_1 = 2.5 \ kN$  and  $P_2 = 3.5 \ kN$
- 2. Deflection,  $\delta = 5 mm$
- 3. Spring Index, C = 5
- 4. Ultimate tensile strength,  $S_{ut} = 1050 MPa$
- 5. Modulus of rigidity, G = 81370 Mpa
- 6. Permissible shear stress,  $\tau = 0.5S_{ut}$

Data Given: Wire diameter • formulae:  $\tau = k \left[ \frac{8 \cdot P \cdot C}{\pi d^2} \right]$ 1. Axial force,  $P_1 = 2.5 \ kN$  and  $P_2 = 3.5 \ kN$ 2. Deflection,  $\delta = 5 mm$  Mean coil diameter 3. Spring Index, C = 5• Formulae: Spring Index =  $\frac{D}{d}$ 4. Ultimate tensile strength,  $S_{ut} = 1050 MPa$ • Number of active coils 5. Modulus of rigidity, G = 81370 Mpa • Formulae: deflection of spring  $\delta = \frac{8PNC^3}{Gd}$ 6. Permissible shear stress,  $\tau = 0.5S_{ut}$  Solid length of spring • Solid length = Total number of coils  $(N_t) \times wire$  diameter (d)• Total number of coils,  $N_t = N + 2$  Free length of spring • Free length of spring = Solid length + deflection Spring rate • Spring rate =  $\frac{P_1 - P_2}{\delta}$ • Actual spring rate • formulae:  $\tau = k \left[ \frac{8 \cdot P \cdot C}{\pi d^2} \right]$ 

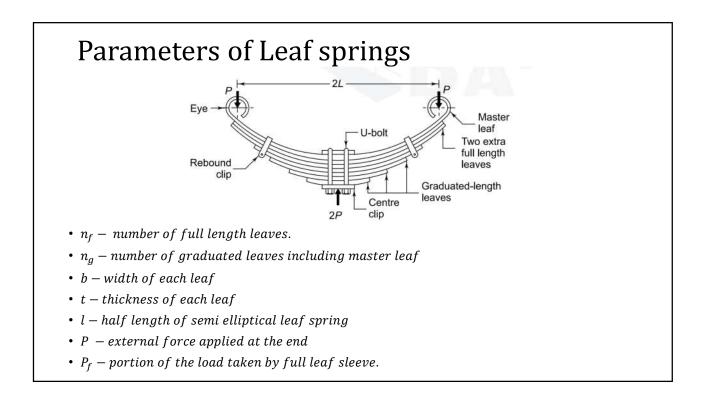
## Surge in Springs

- When the natural frequency of spring coincides with the frequency of external force, resonance occurs.
- During this resonance state, the compression waves travel in forward and return path.
- This type of vibratory motion is called as surge.
- Surge can be observed when the springs are subjected to periodic forces.

• The natural cyclic frequency of helical compression spring that is present between two plates is  $\omega_n = \frac{1}{2}\sqrt{\frac{k}{m}}$ 

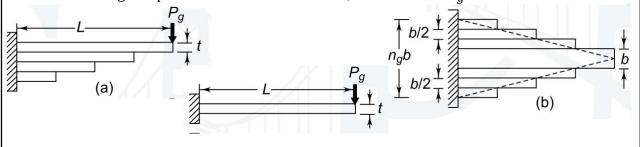
#### Laminated/Leaf springs

- Leaf springs are a basic form of suspension made up of layers of steel of varying sizes sandwiched one upon the other.
- Leaf spring deflects to act as a structural member
- Energy is absorbed while the leaf spring is subjected to shock or impact loads.
- Leaf spring consists of multiple leaf's.
- Each leaf is attached to one another.
- Leaf is in form of a flat plate with semi-elliptical shape.
- The top leaf will have maximum length (full length) and it is called as master leaf.
- There will be 2 or 3 full length leaves which are used for support.



#### Assumption for analysis

- Graduated leaf along with master leaf is considered as a triangular plate.
- Master leaf is placed in the center of the graduated leaves.
- Graduated leafs are cut to half and attached at both up and down sides of master leaf
- The result of this assumption is:
  - Triangular plate will have thickness is t, total width  $= n_g \cdot b$



### Analysis

- Due to the applied loads, bending action will takes place in the leaf spring.
- The bending stress developed in the plate can be calculated by using bending equation.

• Hence 
$$(\sigma_b)_g = \frac{M_b \times y}{I} = \frac{(P_g \times L) \times \frac{t}{2}}{\left(\frac{1}{12} \cdot \left((n_g \cdot b) \cdot t^3\right)\right)} = \frac{6 \cdot P_g \cdot L}{(n_g \cdot b) \cdot t^2}$$

• The deflection created by this force on the assumed triangular component is: •  $\delta_g = \frac{P_g \cdot L^3}{2 \cdot E \cdot I_{max}} = \frac{P_g \cdot L^3}{2 \cdot E \cdot \left(\frac{1}{12} \cdot \left((n_g \cdot b) \cdot t^3\right)\right)} = \frac{6 \cdot P_g \cdot L^3}{E \cdot (n_g \cdot b) \cdot t^3}$ 

• Similarly for extra full length leaves, $(\sigma_b)_f = \frac{M_b \times y}{I} = \frac{(P_f \times L) \times \frac{t}{2}}{\left(\frac{1}{12} \cdot \left((n_f \cdot b) \cdot t^3\right)\right)} = \frac{6 \cdot P_f \cdot L}{(n_f \cdot b) \cdot t^2}$  and defection is  $\delta_f = \frac{P_f \cdot L^3}{3 \cdot E \cdot I_{max}} = \frac{P_f \cdot L^3}{3 \cdot E \cdot \left(\frac{1}{12} \cdot \left((n_f \cdot b) \cdot t^3\right)\right)} = \frac{4 \cdot P_f \cdot L^3}{E \cdot (n_f \cdot b) \cdot t^3}$ 

## Analysis

- Since the deflection for the whole leaf spring is same, hence  $\delta_g=\delta_f$ 
  - Which means,  $\frac{6 \cdot P_g \cdot L^3}{E \cdot (n_g \cdot b) \cdot t^3} = \frac{4 \cdot P_f \cdot L^3}{E \cdot (n_f \cdot b) \cdot t^3}$

• 
$$\frac{s}{P_f} = \frac{1}{3 \cdot n}$$

- Also total force  $P = P_g + P_f$
- Therefore the magnitudes of forces shared by graduated leaves and full length leaves are:

• 
$$P_f = \frac{3 \cdot n_f \cdot P}{(3 \cdot n_f + 2 \cdot n_g)}$$
 and  $P_g = \frac{2 \cdot n_g \cdot P}{(3 \cdot n_f + 2 \cdot n_g)}$ 

• Substituting the above values in bending stresses, we get:

• 
$$(\sigma_b)_g = \frac{12PL}{(3\cdot n_f + 2\cdot n_g)\cdot bt^2}$$
 and  $(\sigma_b)_f = \frac{18PL}{(3\cdot n_f + 2\cdot n_g)\cdot bt^2}$ 

• The deflection at the end leaf of the spring is  $\delta = \frac{12PL^3}{(3 \cdot n_f + 2 \cdot n_g) \cdot Ebt^3}$ 

### Nipping of springs

• In the above analysis it is observed that, the stresses in full length leaves and graduated leaves are:

•  $(\sigma_b)_g = \frac{12PL}{(3 \cdot n_f + 2 \cdot n_g) \cdot bt^2}$  and  $(\sigma_b)_f = \frac{18PL}{(3 \cdot n_f + 2 \cdot n_g) \cdot bt^2}$ 

- It can be observed that the full length leaves are subjected to 50% more stresses than the graduated leaves
- Due to this failure chances are more.
- So to avoid this, initially pre-stresses are induced to equalize the stresses in both full length and graduated leaves.
- Pre-stressing is achieved by bending the leaves to a different radius of curvature.
- Due to this bending of leaves to induce pre-stresses, a small extra gap will be created between the full length and graduated leaves.
- This gap is called as *nip*. And the process of achieving pre-stresses by difference in the radius of curvature is called *nipping*.

#### Analysis - Nipping

- So if nipping is being done, then both the stresses for full length leaves equal to graduated leaves.
- We know that the bending stress in full length and graduated leaves are: •  $(\sigma_b)_g = \frac{6 \cdot P_g \cdot L}{(n_g \cdot b) \cdot t^2}$  and  $(\sigma_b)_f = \frac{6 \cdot P_f \cdot L}{(n_f \cdot b) \cdot t^2}$
- So assuming pre-stressing results in equalizing of the stresses

• 
$$(\sigma_b)_g = \frac{6 \cdot P_g \cdot L}{(n_g \cdot b) \cdot t^2} = (\sigma_b)_f = \frac{6 \cdot P_f \cdot L}{(n_f \cdot b) \cdot t^2}$$

• This results in  $\frac{P_g}{P_f} = \frac{n_g}{n_f}$  and total force  $P = P_g + P_f$  and total leaves,  $n = n_g + n_f$ 

• Using the above equations we get, 
$$P_g = \frac{n_g \cdot P}{n}$$
 and  $P_f = \frac{n_f \cdot P}{n}$ 

• The deflections in full length leaves and graduated leaves are:

• 
$$\delta_g = \frac{6 \cdot P_g \cdot L^3}{E \cdot (n_g \cdot b) \cdot t^3}$$
 and  $\delta_f = \frac{4 \cdot P_f \cdot L^3}{E \cdot (n_f \cdot b) \cdot t^3}$ 

## Analysis - Nipping • Forces on leaves are: $P_g = \frac{n_g \cdot P}{n}$ and $P_f = \frac{n_f \cdot P}{n}$ • The deflections in full length leaves and graduated leaves are: • $\delta_g = \frac{6 \cdot P_g \cdot L^3}{E \cdot (n_g \cdot b) \cdot t^3}$ and $\delta_f = \frac{4 \cdot P_f \cdot L^3}{E \cdot (n_f \cdot b) \cdot t^3}$ • So as per definition of nipping, it is the difference between the deflection in leaves or initial gap between the leaves. • Hence nipping, $C = \delta_g - \delta_f = \frac{6 \cdot P_g \cdot L^3}{E \cdot (n_g \cdot b) \cdot t^3} - \frac{4 \cdot P_f \cdot L^3}{E \cdot (n_f \cdot b) \cdot t^3}$ • Substituting the values of $P_g$ and $P_f$ in the above equation: • Nipping, $C = \frac{2PL^3}{Enbt^3}$

#### Analysis - Nipping

• To close this gap between the leaves, the magnitude of pre-load (*P<sub>i</sub>*) to be applied is:

• C = 
$$(\delta_g)_i + (\delta_f)_i$$
  
• C =  $\frac{2PL^3}{Enbt^3} = \frac{6 \cdot P_g \cdot L^3}{E \cdot (n_g \cdot b) \cdot t^3} + \frac{4 \cdot P_f \cdot L^3}{E \cdot (n_f \cdot b) \cdot t^3}$ 

- Where the total pre-load is shared by both the full length and graduated leaves
  - Hence,  $P_g = P_f = \frac{P_i}{2}$
- Substituting these values in the above equation,

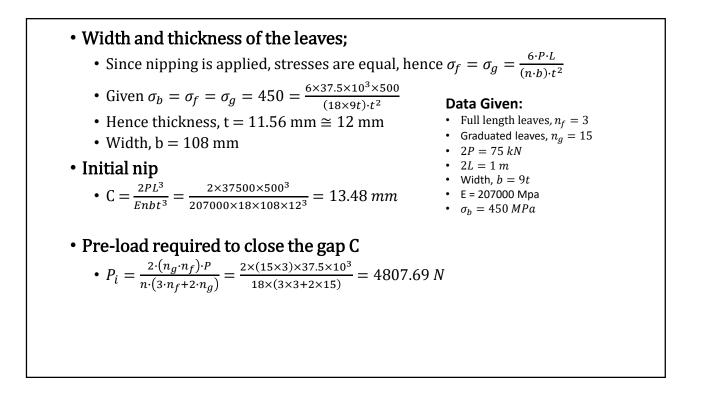
• 
$$\frac{2PL^3}{Enbt^3} = \frac{6 \cdot \frac{P_i}{2} \cdot L^3}{E \cdot (n_g \cdot b) \cdot t^3} + \frac{4 \cdot \frac{P_i}{2} \cdot L^3}{E \cdot (n_f \cdot b) \cdot t^3}$$
  
• 
$$P_i = \frac{2 \cdot (n_g \cdot n_f) \cdot P}{n \cdot (3 \cdot n_f + 2 \cdot n_g)}$$

#### Analysis - Nipping

- So the resultant stresses in the full length leaves are:
  - Before nipping, the stresses are,  $(\sigma_b)_f = \frac{6 \cdot P_f \cdot L}{(n_f \cdot b) \cdot t^2}$
  - When nipping due to the pre-stresses the bending stress is,  $\left((\sigma_b)_f\right)_i = \frac{6\cdot\frac{1}{2}L}{(n_f\cdot b)\cdot t^2}$
- So the resultant stress is  $= (\sigma_b)_f ((\sigma_b)_f)_i = \frac{6 \cdot P_f \cdot L}{(n_f \cdot b) \cdot t^2} \frac{3 \cdot P_i \cdot L}{(n_f \cdot b) \cdot t^2}$
- But we know  $P_f = \frac{3 \cdot n_f \cdot P}{(3 \cdot n_f + 2 \cdot n_g)}$  and  $P_i = \frac{2 \cdot (n_g \cdot n_f) \cdot P}{n \cdot (3 \cdot n_f + 2 \cdot n_g)}$
- Substituting them in the above equation, we get,
- Resultant stresses in full length leaves as,  $\sigma_f = \frac{6 \cdot P \cdot L}{(n \cdot b) \cdot t^2}$
- Since due to nipping, stresses are equal, hence  $\sigma_f = \sigma_g = \frac{6 \cdot P \cdot L}{(n \cdot b) \cdot t^2}$

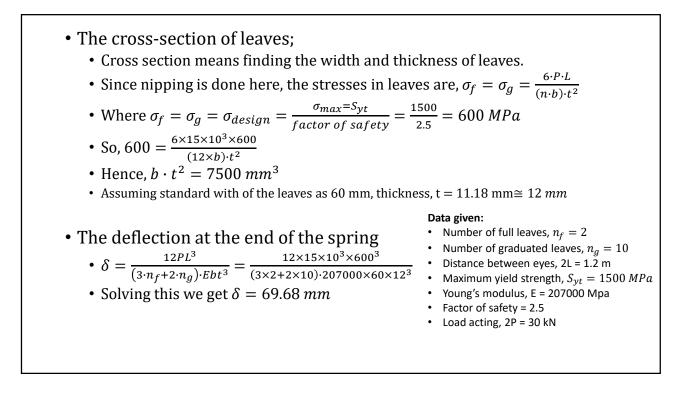
A semi-elliptic leaf spring used for automobile suspension consists of three extra fulllength leaves and 15 graduated-length leaves, including the master leaf. The centre-tocentre distance between two eyes of the spring is 1 m. The maximum force that can act on the spring is 75 kN. For each leaf, the ratio of width to thickness is 9:1. The modulus of elasticity of the leaf material is 207000 N/mm<sup>2</sup>. The leaves are pre-stressed in such a way that when the force is maximum, the stresses induced in all leaves are same and equal to 450 N/mm<sup>2</sup>. Determine

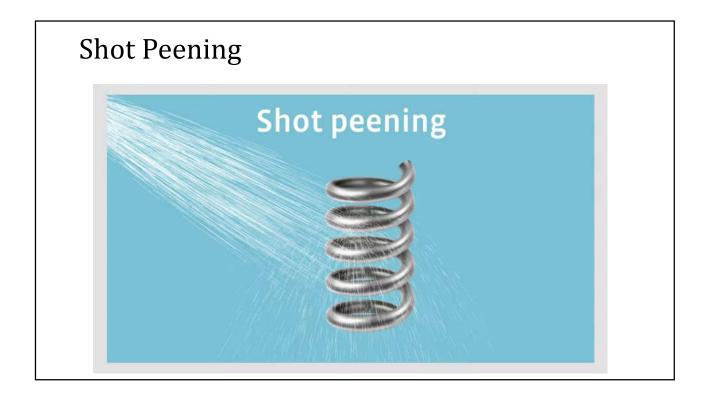
- 1. the width and thickness of the leaves;
- 2. the initial nip; and
- 3. the initial pre-load required to close the gap C between extra full-length leaves and graduated-length leaves.
- Data Given:
  - Full length leaves,  $n_f = 3$
  - Graduated leaves,  $n_g = 15$
  - $2P = 75 \ kN$
  - 2L = 1 m
  - Width, b = 9t
  - E = 207000 Mpa
  - $\sigma_b = 450 MPa$



A semi-elliptic multi-leaf spring is used for the suspension of the rear axle of a truck. It consists of two extra full-length leaves and ten graduated-length leaves including the master leaf. The centre-to-centre distance between the spring eyes is 1.2 m. The leaves are made of steel 55Si2Mo90 ( $S_{yt} = 1500$  N/mm<sup>2</sup> and E = 207000 N/mm<sup>2</sup>) and the factor of safety is 2.5. The spring is to be designed for a maximum force of 30 kN. The leaves are pre-stressed so as to equalize stresses in all leaves. Determine

- The cross-section of leaves; and
- The deflection at the end of the spring
- Data given:
  - Number of full leaves,  $n_f = 2$
  - Number of graduated leaves,  $n_g = 10$
  - Distance between eyes, 2L = 1.2 m
  - Maximum yield strength,  $S_{yt} = 1500 MPa$
  - Young's modulus, E = 207000 Mpa
  - Factor of safety = 2.5
  - Load acting, 2P = 30 kN





#### Shot peening

- It is observed that failure in the springs is mostly due to the fluctuating stresses
- This is due to the poor surface finish which results in crack propagation.
- Whereas, to avoid this crack propagation, the surface need to be strengthened with a layer of residual stresses on the surface.
- The method of creating such layer of residual stresses is called shot peening.
- In this process, small steel balls are impinged on the wire surface with high velocity.
- The balls strike the surface of the wire and induce residual compressive stresses.
- Shot peening is more effective for helical springs.