UNIT-3

BALANCING OF RECIPROCATING MASSES

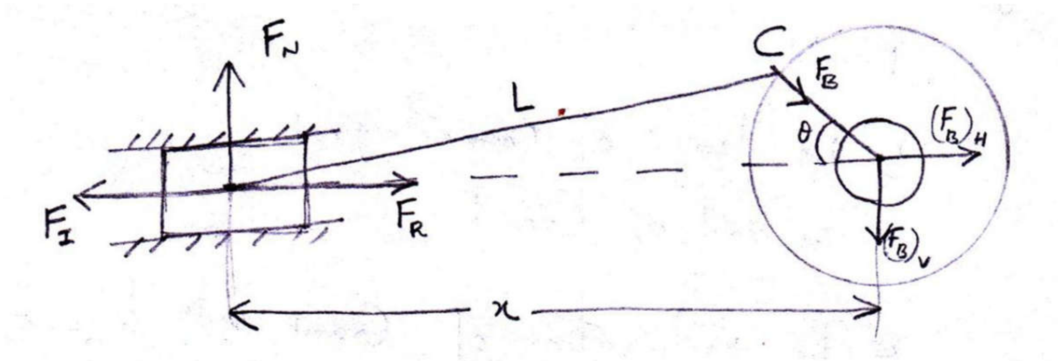
Syllabus

- Introduction
- Primary and Secondary Unbalanced forces of reciprocating masses
- Partial balancing of unbalanced primary forces in an engine
- Effect of partial balancing of reciprocating parts of 2 cylinder locomotives
- Variation of tractive forces
- Swaying couple and Hammer blow

Introduction

- So in our previous topic we discussed about balancing of rotating components.
- From that topic we understood that rotating components need to be balanced, else they will cause disturbance to system.
- But does the reciprocating masses need to balanced..?
- What happens if the reciprocating mass is not balanced..?
- Where can we find such situation..?
- Situation:
  - Generally in Engine various forces will be generated in different directions, so some forces will be used as a useful work where as the other forces will try to pull the system as per their direction
  - So these forces are called as unbalanced reciprocating forces.

Analysis of Engine Reciprocating forces



- During the working of Engine, the following forces acts on the system:
  - Piston effort
  - Force acting along the connecting rod
  - Thrust on the cylinder walls
  - Thrust on the crank shaft bearing

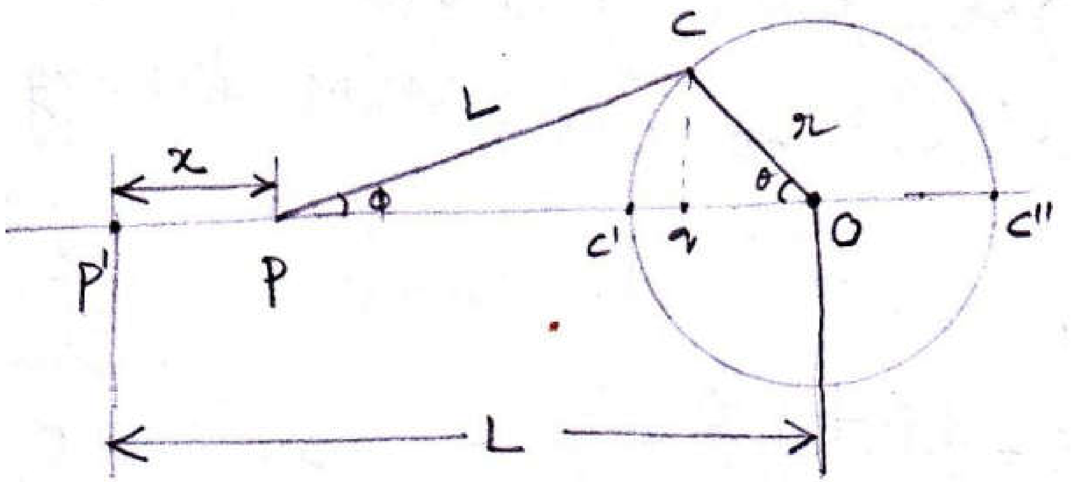
Analysis of Engine Reciprocating forces

- During expansion Process in an engine, the gas force inside the cylinder chamber exerts pressure on the piston in all directions.
- The piston will transfer some of this gas force in the path of connecting rod and through connecting rod to crank and from there to crank shaft and so on..
- So the remaining magnitude of this gas force will be used to reciprocate the piston inside the cylinder chamber along the horizontal direction.
- So in order to move the piston in horizontal direction, the external force should be equal to the inertia force (hence $F_I = F_R$)
- The force from connecting rod to crank is an inclined force which will be acted on crank bearings (F_B)

Analysis of Engine Reciprocating forces

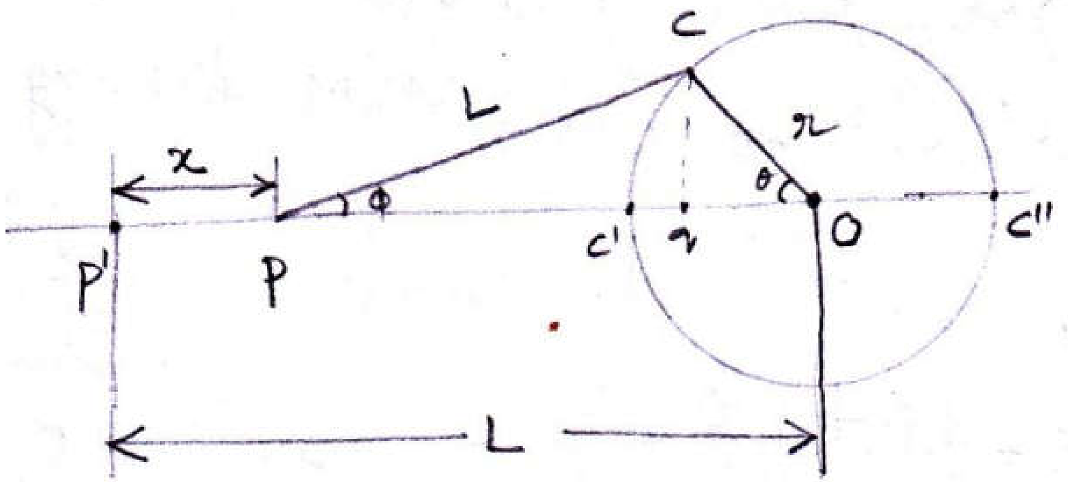
- Bearing Forces:
  - As discussed the bearing force is inclined, hence this force can be resolved in to two components (i.e F_{B_H} and F_{B_V})
- The piston and the cylinder have a surface contact between them, hence during relative motion, there will be a friction force and normal reaction exists between them.
- As the friction force created a serious damage to the cylinder lining, hence proper lubrication is provided thereby ensuring that this frictional force doesn't create damage to system. Therefore it can be neglected.
- Where as the remaining forces present in the system which are not yet balanced are:
  - Bearing forces (i.e F_{B_H} and F_{B_V})
  - Normal reaction (F_N) at the surface contact of piston & cylinder.
- It is observed experimentally that F_{B_V} and F_N are in opposite direction and have a certain distance between them, so they constitute a couple.
- **So in a nut shell for the total system we have a unbalanced force (F_{B_H}) and an unbalanced couple (due to F_{B_V} and F_N)**

Primary and Secondary unbalanced forces of Reciprocating mass



- Consider the following single slider mechanism of an engine:
  - Let m be mass of reciprocating components.
  - L-length of connecting rod, r-radius of crank
  - θ-angle made by crank with line of stroke, ω-angular velocity of crank
  - n-ratio of length of connecting rod to radius of crank.
- Aim: to find the magnitudes of primary and secondary forces as a function of independent variable.

Primary and Secondary unbalanced forces of Reciprocating mass



- Assumptions:
  - Initial position of the crank is c^I
  - Crank rotates at ω rad/s in clockwise direction
  - Let c be the new position of the crank now which makes an angle θ with line of stroke.
- Due to this crank rotation the position of piston moves from P^I to P, making a displacement of '*x*'

Primary and Secondary unbalanced forces of Reciprocating mass

- Algebric Derivation:

From the diagram $x = PP^I = OP^I - OP$

$x = OP^I - OP = (P^I C^I + C^I O) - (PQ + OQ)$

But we know that $P^I C^I = L$ *and* $C^I O = r$, *from the triangle* $PQ = L \cos \Phi$ *and* $OQ = r \cos \theta$

So $x = (L + r) - (L \cos \Phi + r \cos \theta) = L (1 - \cos \Phi) + r (1 - \cos \theta)$

Taking radius as common for whole equation

$x = r\left[\frac{L}{r}(1 - \cos \Phi) + (1 - \cos \theta)\right] = r\,[n\,(1 - \cos \Phi) + (1 - \cos \theta)] \rightarrow$ (1)

In the above equation we have two input independent variables (θ and Φ), where as we only need one controlling parameter in our expressions, hence we select θ

From the ΔCPQ and ΔCQO the common side is CQ, so from right angle triangle rule

$CQ = L \sin \Phi = r \sin \theta \rightarrow \sin \Phi = \frac{r}{L} \sin \theta = \frac{\sin \theta}{n}$

But in eq^n *1 we have* $\cos \Phi$ *so changing the* $\sin \Phi$ *into* $\cos \Phi$

$\cos \Phi = \sqrt{1 - \sin^2 \Phi} = \sqrt{1 - \left(\frac{\sin \theta}{n}\right)^2} = 1 - \frac{1}{2}\frac{\sin \theta^2}{n^2}$ *(as per taylor's expansion avoiding the higher order terms in the expansion)*

Substituting the above cos Φ in equation 1

$x = r\left[(1 - \cos \theta) + \frac{1}{2}\frac{Sin^2 \theta}{n}\right] \rightarrow$ (2) *(this is the displacement* eq^n *of the piston w.r.t θ)*

Primary and Secondary unbalanced forces of Reciprocating mass

- So we had obtained the expression for displacement change due to change in crank orientation.
- But we are interested in finding the forces here..
- So let us use newton 2nd law
  - Force = mass × acceleration
    - Since we are doing analysis here Let us change the formula, Force = mass × $\frac{dV}{dt}$
    - So $\frac{dV}{dt}$ represent change in velocity w.r.t time and velocity = $\frac{dx}{dt}$
    - Since we had found the displacement expression we can find the velocity and thereby acceleration.
  - Velocity = $\frac{dx}{dt} = \frac{d\left(r\left[(1 - cos\ \theta) + \frac{1}{2}\frac{Sin^2\theta}{n}\right]\right)}{d\theta} \times \frac{d\theta}{dt}$
  - But we know $\frac{d\theta}{dt}$ is angular velocity (ω).
  - So velocity = ω.r $\left(\sin\theta + \frac{\sin 2\theta}{2n}\right)$
- Similarly acceleration = $\frac{dV}{dt} = \frac{dV}{d\theta} \times \frac{d\theta}{dt}$ = ω^2.r $\left(\text{Cos}\,\theta + \frac{\text{Cos}\,2\theta}{n}\right)$

Primary and Secondary unbalanced forces of Reciprocating mass

- So as per the obtained expression for acceleration we can find the magnitude of total force.
  - Force = m.r.ω^2 $\left(\text{Cos}\,\theta + \frac{\text{Cos}\,2\theta}{n}\right)$
- So what is this force…? Let's discuss about it.
- So how many strokes does an engine will have? And what are they?
  - Suction stroke, Compression Stroke, Expansion stroke, Exhaust.
  - So out of these, expansion stroke is the power stroke which gives the driving force for the engine.
  - So how the remaining strokes are going to work then..?
  - Let us answer this question by a small example
    - If you are driving a bike at some high speed, can you able to stop all of sudden..?
      - No, it will take few second even if you apply brakes hardly.
    - Why..?
      - The reason for this is inertia of the automobile.
  - So the inertia force will be present everywhere, hence this inertia of the engine weight components created during the power stroke will be responsible for the engine to work during non power strokes i.e. during suction, compression, exhaust.

Primary and Secondary unbalanced forces of Reciprocating mass

- So as per our discussion, it is evident that the inertia force is responsible to create movement/motion to the components of the engine during non power stroke.
  - So what is the nature of motion happened inside an engine cylinder w.r.t piston?
    - Reciprocating motion between Piston and cylinder.
  - Therefore $F_I = F_R = m.r.\omega^2 \left(\text{Cos}\,\theta + \frac{\text{Cos}\,2\theta}{n}\right)$
  - Earlier during our discussion about the force analysis in engine, we discussed about bearing force, the horizontal component of bearing force is equated to inertia force.
    - So $F_{B_H} = F_I$ ---→ why..?
  - So $F_{B_H} = F_I = m.r.\omega^2 \left(\text{Cos}\,\theta + \frac{\text{Cos}\,2\theta}{n}\right)$ → remember that this discussion is happening during non power stroke only.
  - The above expression can we written as $F_{B_H} = m.r.\omega^2\,(\text{Cos}\,\theta) + m.r.\omega^2 \left(\frac{Cos\,2\theta}{n}\right)$
    - Where Primary unbalanced force = $m.r.\omega^2\,(\text{Cos}\,\theta)$
    - Secondary unbalanced force = $m.r.\omega^2 \left(\frac{Cos\,2\theta}{n}\right)$

Partial balancing of unbalanced primary forces in an engine

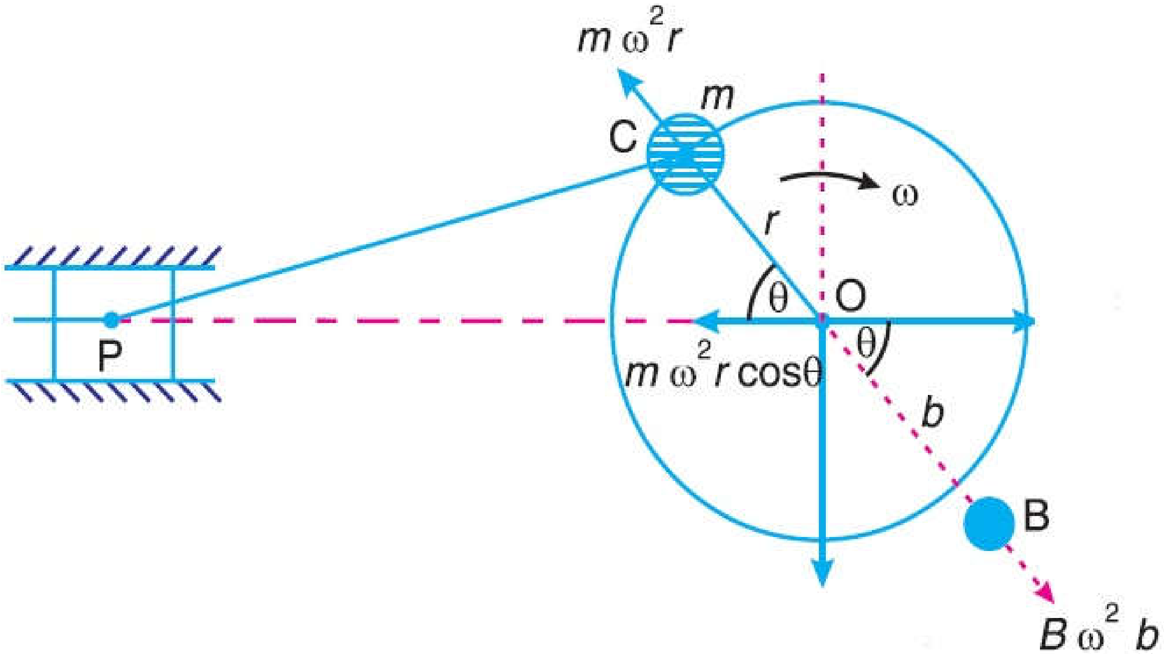
- So in the previous topic we discussed about the magnitudes of the primary and secondary reciprocating forces.
- These reciprocating forces will create unbalance to the system.
- Now we should balance this unbalance reciprocating forces.
- So From the title of the topic we should get some doubts..
  - Why “Partial Balancing”…?
    - Why only partial balancing, cant we able to do complete balancing of this unbalanced forces.?
  - No, Complete balancing of system which have motion is quite difficult
    - Since for every second they change their position and orientation. Therefore the only way to reduce the disturbance is by partial balancing.
- Why partial balancing of primary forces only..?
  - In the whole magnitude of the unbalanced forces, the major portion will be contributed by primary forces only. Hence primary forces are our topic of interest.

Partial balancing of unbalanced primary forces in an engine

- So how to do partial balancing to this system..?
- In our previous chapter we learnt how to balance the unbalanced rotating forces.
- Now can we use the same methods to balance this reciprocating unbalanced forces..?
  - If we convert the unbalanced reciprocating forces as unbalanced rotating force, then the methods of balancing the rotating forces can be used.
- So how can we do this conversion..?
- If we have a rotating unbalanced mass, which kind of force it creates..?
  - Centrifugal force with a magnitude of $F_C = m.r.\omega^2$
- If we observe the magnitude of unbalanced primary reciprocating force
  - $F_P = m.r.\omega^2 \ (\text{Cos}\, \theta) = F_C \text{ Cos } \theta$.
- $F_C \cos\theta$ is like a component of an inclined force. So this is the assumption we will consider here.

Partial balancing of unbalanced primary forces in an engine

- Assumption:
  - Imagine that there is a mass (m) placed on rotating component of the engine at a distance of radius (r) making an angle θ w.r.t line of stroke.
    - Which component in engine have rotational motion..?
      - Crank
    - So we re placing the imaginary mass on the crank which rotated with angular velocity ω rad/s



Partial balancing of unbalanced primary forces in an engine

- The very important thing here to understand is there is no mass on the crank, we are just imagining it.
  - Now if we have a unbalanced mass on crank which is rotating at ω rad/s, this creates a centrifugal force of magnitude $F_C = m.r.\omega^2$
  - This force is an inclined force since crank is inclined so if we resolve into components, we get
    - $F_{C_H} =$ m.r.ω² cos θ. And $F_{C_V} =$ m.r.ω² sin θ ------> Just imaginary forces only
- Remember again that we are assuming an imaginary mass on the crank, so the forces are also imaginary here.
- If we observe the imaginary force (F_{C_H}) coincides with the magnitude of primary unbalanced reciprocating force.
- Proposed solution:
  - If we can able to balance the imaginary force (F_{C_H}) then intuitively we balance the primary unbalanced reciprocating force then.

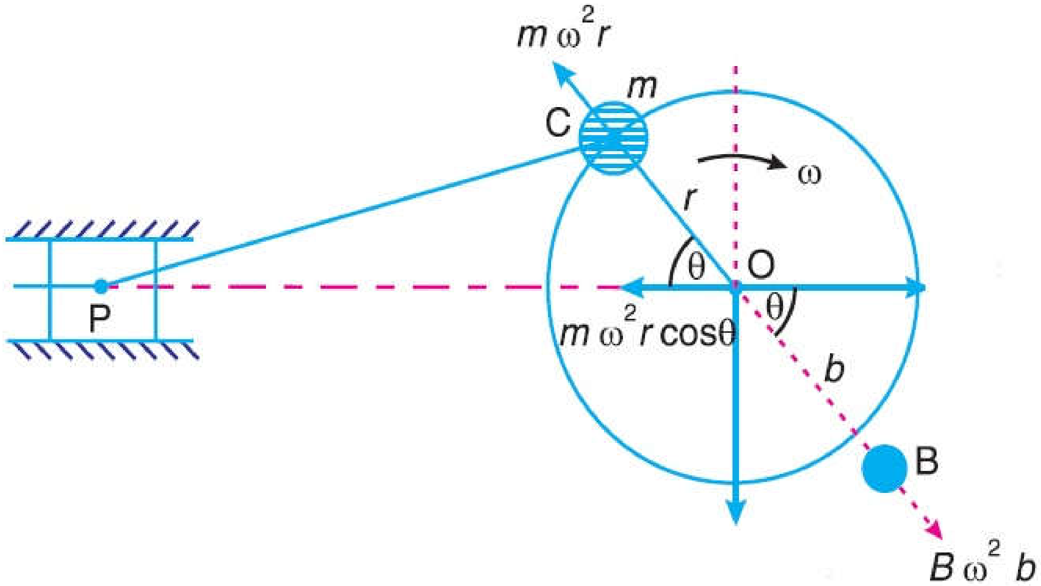
Partial balancing of unbalanced primary forces in an engine

- So as per our discussion, it is understood that now in order to balance the primary force we should balance the horizontal component of imaginary force.
- So now this problem becomes Rotating balance problem, since imaginary force is due to the imaginary mass on the crank which is a rotating component.
- What different cases of balancing did we learnt in Balancing of Rotating masses concept..?

No of masses to be Balanced	No of masses balancing these unbalanced masses	Balancing masses plane (same plane/different plane)
one	one	Same plane
one	Two	Different plane
Many	one	Same plane
Many masses rotating in different planes	Two	Different

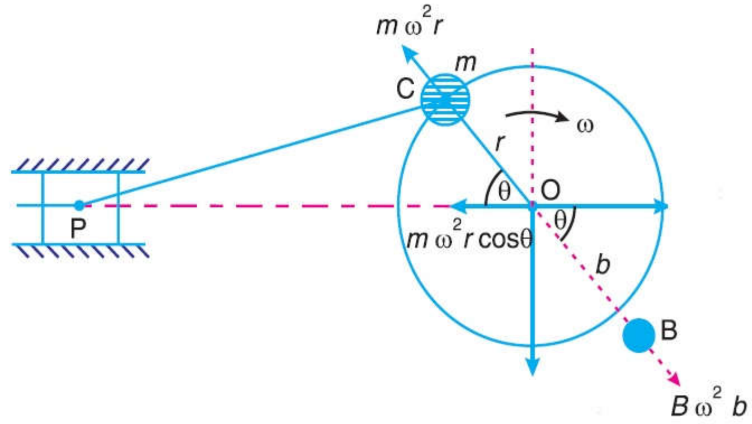
Partial balancing of unbalanced primary forces in an engine

- So now we had chosen our method to balance this imaginary force
  - So we learnt that in order to balance a single unbalanced mass by using a single balancing mass in same plane.
- Solution:
  - To do that we need to install another balancing mass on the same disc (plane) exactly opposite side.



Partial balancing of unbalanced primary forces in an engine

- So let B be the balancing mass attached on the crank, let b is its radius of rotation and θ its orientation.
- So this B mass is rotating hence creates centrifugal force.
  - $[F_C]_B$ = B.b.ω^2
  - This force is again inclined force, so resolving to components produces:
    - $([F_C]_B)_H$ = B.b.ω^2 cos θ
    - $([F_C]_B)_V$ = B.b.ω^2 sin θ
- So the primary unbalanced force is horizontal component (left side) and this $([F_C]_B)_H$ is also horizontal component (right side)



Therefore in order to have the primary unbalanced force to get balanced:

$F_C = ([F_C]_B)_H$

m.r.ω^2 cos θ = B.b.ω^2 cos θ

So, m.r = B.b is the condition which is to be satisfied.

Analysis: Important Case i

- So from the above concept of balancing the primary reciprocating unbalanced force there are few important things on which we should be clear about:
  - Does the primary unbalanced force got balanced..?
    - yes
  - Is the arrangement of using a balancing mass (B) on crank created any side effects..?
    - Yes
  - So what sort of side effects are those..?
    - So the balancing mass during its rotation creates which force
      - Centrifugal Force $(F_C)_B$ which is inclined in nature
      - So what does the component forces of $(F_C)_B$ do..?
        - $([F_C]_B)_H$ is balancing the primary reciprocating unbalanced force F_P
        - Where as $([F_C]_B)_V$ is still creating disturbance.
      - So due to our arrangement of using balancing mass (B), what happens finally..?
        - Plus side is Primary reciprocating unbalanced force is balanced.
        - Negative side is Another unwanted force i.e. $([F_C]_B)_V$ is being generated.
- Finally the force remained in the system is $([F_C]_B)_V = B.b.\omega^2 \sin\theta$
- So even though primary reciprocating unbalanced force is balanced, creating this unwanted force is side effect, hence the system is not completely balanced. So the title is given as *"partial balancing"*.

Analysis:

- So in the above analysis we had discussed few important points:
  - Primary reciprocating unbalanced force is balanced.
  - An unwanted force i.e. $([F_C]_B)_\mathrm{V}$ is being generated.
- So in our above conclusions, let us consider a new case ii:
  - What if the primary reciprocating unbalanced force is not completely balanced..?
  - This case is called as partial balancing of ***Primary reciprocating unbalanced force***.
  - So now what are we interested..?
    - To find the Total resultant force acting on the system.
- Situation:
  - In an engine, there are some unbalanced forces present and creating disturbance. It is identified that these unbalanced forces are categorized in to two types: Primary and secondary.
  - Now to balance this primary reciprocating unbalanced force, a balancing mass B is attached on the crank. But upon observation it is identified that the primary reciprocating unbalanced force is not completely balanced.

Analysis: Important Case ii

- Aim:
  - As the system is not completely balanced even though a balancing mass is used we need to find the resultant force acting on the system.
- Assumption:
  - Let us assume that C be the fraction of the primary reciprocating unbalanced force which got balanced due to the added mass B on the crank.
    - i.e if the primary force is completely balanced, the condition is **m.r = B.b**
    - Similarly if only fraction of primary force is balanced, then condition is modified as
      - **B.b = C.m.r**
- Solution:
  - Horizontal Forces acting in the system are: F_P and $([F_C]_B)_H$
  - Vertical Forces acting in the system are: $([F_C]_B)_V$
- Condition: Primary force is partially balanced.

Analysis: Important Case

- Total Horizontal Force $F_H = F_P - ([F_C]_B)_H$ = m.r.ω^2 cos θ – B.b.ω^2 cos θ
  - But since C is the fraction which by which this primary force is balanced, so **B.b = C.m.r**
  - $F_H = (1 - C)$ m.r.ω^2 cos θ
- Total vertical force $F_V = ([F_C]_B)_V$ = B.b.ω^2 sin θ = C.m.r. ω^2 sin θ
- Total Resultant unbalanced force acting on the system = $\sqrt{F_H^2 + F_V^2}$
  - F = (m.r.ω^2) $\sqrt{(1 - C)^2 Cos^2\theta + C^2 Sin^2\theta}$

Effects of Partial balancing of locomotives

- What is a locomotive..?
  - A locomotive is a mechanical device which is used to create displacement for a system.
- Usually locomotives are of two types-
  - Coupled locomotives
  - Uncoupled locomotives.
- In case of an engine system we understood that only partial balancing is possible, so we need to discuss about the effects of the partial balancing in locomotives.
- If you consider a planar system, there will be X-axis and y-axis present.
- Similarly the unbalanced forces acts in both X and Y directions. Where as here we consider line of stroke as reference.

Effects of Partial balancing of locomotives

- So the unbalanced forces acting on the engine in locomotive will have certain effects in two directions:
  - Along the line of stroke
    - Variation of Tractive force
    - Swaying Couple
  - Perpendicular to line of stroke.
    - Hammer Blow
- Tractive force:
  - Note: In case of a 2 cylinder locomotives there will be 2 cranks aligned in such a way that they will be perpendicular w.r.t each other.
  - The resultant unbalanced force for a locomotive along the line of stroke is known as Tractive force.
  - Let us assume that the locomotive has 2 cylinders where for the first cylinder θ is the angle made by crank and then for the second cylinder the angle will be $90+ \theta$.

Effects of Partial balancing of locomotives

- From our previous discussion of "Partial balancing of unbalanced primary forces" we had obtained the expression for horizontal component of unbalanced primary force [since horizontal component represents line of stroke]
  - $F_H = (1 - C)\ m.r.\omega^2 \cos\theta$
- So for the 2 cylinders the unbalanced primary force is as follows:
  - For first cylinder = $F_{H_1} = (1 - C)\ m.r.\omega^2 \cos\theta$
  - For second cylinder = $F_{H_2} = (1 - C)\ m.r.\omega^2 \cos(90+\theta)$
- Hence the tractive force for the complete system = $F_{H_1} + F_{H_2}$
  - Tractive force $F_T = (1 - C)\ m.r.\omega^2\ [\cos\theta - \sin\theta]$
- Special case : Maximum Tractive force $(F_T)_{max}$
  - This occurs when the θ is 135^0 or 315^0
  - $(F_T)_{max} = \pm\sqrt{2}\,(1 - C)\ m.r.\omega^2$

Effects of Partial balancing of locomotives

- Swaying Couple:
  - In the previous section the unbalanced forces along the line of stroke by the 2 engine cylinders of the locomotive is found and termed as tractive force.
  - Now we assumed that for a locomotive there will be 2 cylinders and their respective cranks are perpendicular to each other.
  - Let us also assume that there is a distance between them and this distance be “a”.
  - Since we have two unbalanced forces (F_{H_1} and F_{H_2}) acting along line of stroke but have a distance between them, they form a torque (we call here as couple)
  - So the magnitude of this swaying torque is = $F_{H_1} \times \frac{a}{2} - F_{H_2} \times \frac{a}{2}$

- Swaying Torque = $(1 - C)\ m.r.\omega^2\ [\cos\theta + \sin\theta] \times \frac{a}{2}$
- Maximum value of swaying torque = $\pm (1 - C)\ m.r.\omega^2 \times \frac{a}{\sqrt{2}}$

Effects of Partial balancing of locomotives

- Hammer Blow:
  - This is the maximum magnitude of unbalanced force acting perpendicular to line of stroke.
  - So from the "Partial Balancing of unbalanced primary force" topic we had found the magnitude of the unbalanced vertical force acting on the engine.
    - $F_V = ([F_C]_B)_V = B.b.\omega^2 \sin\theta$
  - This vertical force represents the force which is perpendicular to line of stroke.
  - Hence Hammer blow = Max. of F_V
  - The maximum value will be obtained when the angle θ is 90^0
  - Hence Hammer blow = $B.b.\omega^2$

Problem:

An inside cylinder locomotive has its center line 0.7m apart and has a stroke of 0.6m apart. The rotating masses per cylinder are equivalent to 150 kgs at crank pin and reciprocating masses per cylinder are equal to 180 kgs. The wheel lines are 1.5m apart, which are used to balance the system. The cranks are at right angles.

The whole of the rotating mass and $\frac{2}{3}$ of reciprocating mass are to be balanced by masses placed at radius of 0.6m. Find the magnitude and direction of balancing masses.

Find the magnitude of variation in tractive force and swaying couple at a crank speed of 300 rpm.