UNIT-2

TOOTHED GEARS AND GEAR TRAINS

- Course Objective:
  - To make the students understand about gears and gear train configurations.
- Course Outcome:
  - *Solve practical problems related to gears and gear trains in industries*

Syllabus

Toothed Gearing:

- Classification of toothed wheels
- Terminolgy
- Condition for constant velocity ratio
- Law of gearing-velocity of sliding of teeth
- Forms of tooth
- Length of contact, arc of contact
- Interference in involute gears
- Minimum number of tooth to avoid interference

Introduction

- Why use gears?
  - For power transmission.
- I think we have many power transmission systems like ropes, pulleys, chains etc.. So why exactly do we need these gears..?
  - So to get an answer for this let us discuss the pro’s and con’s
- Advantages:
  - Can transmit exact velocity ratio.
  - Large power can be transmitted with very minimum power loss.
  - Compact layout
  - Highly reliable
  - Robust system
- Disadvantages:
  - Cost of components is high
  - Error in manufacturing creates additional problems to the system.

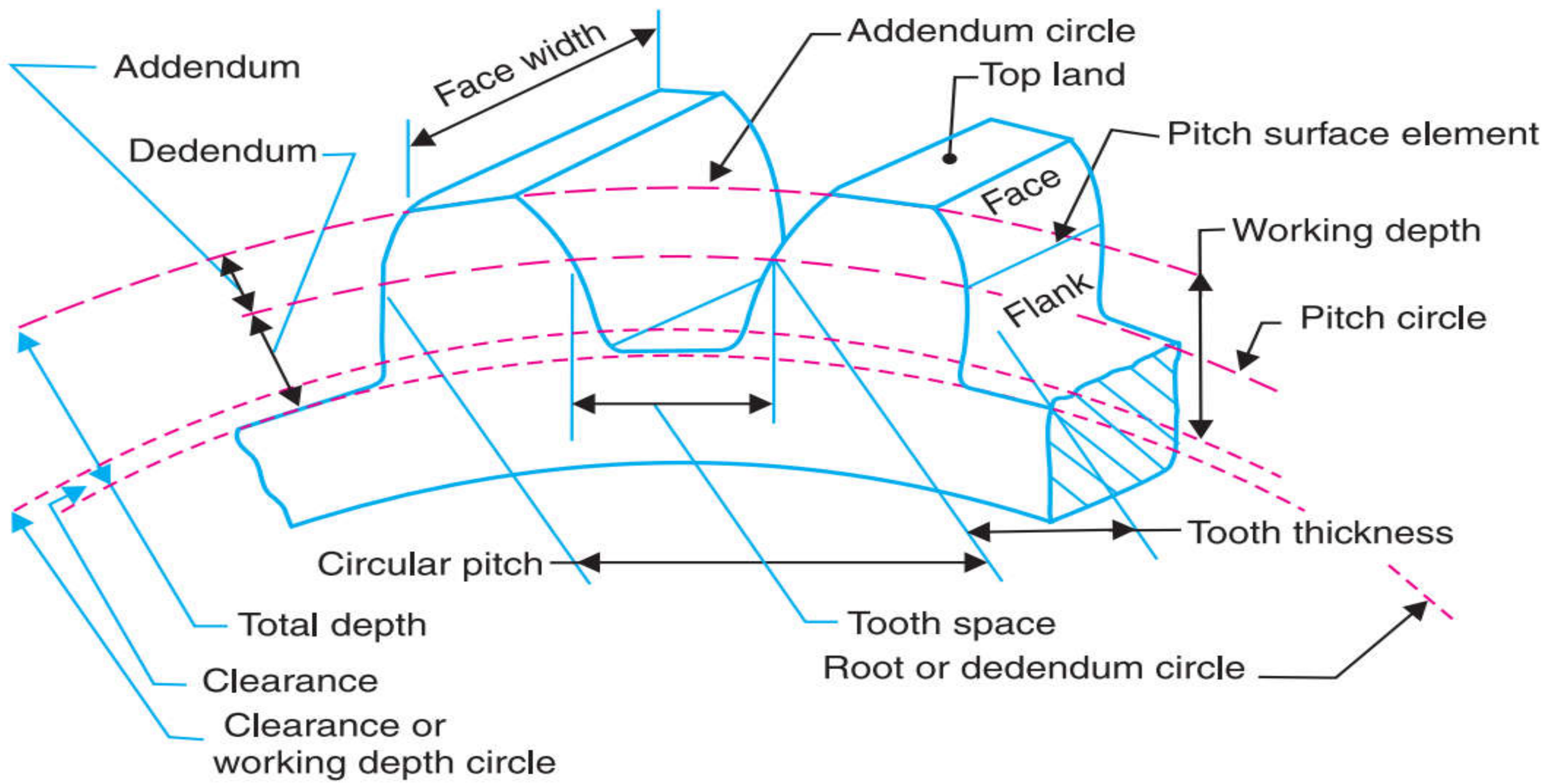
Working & Classification of gears

Gear classification is based on the following factors -

- Position of shaft axis
  - Parallel Axis Gears – Spur gear
  - Intersecting Axis gears – Bevel Gear
  - Non intersecting and non parallel axis gears – Spiral gear
- Peripheral Velocity of the gears
  - Low Velocity gears ($V < 3$ m/s)
  - Medium Velocity gears ($3 < V < 15$ m/s)
  - High Velocity gears ($V > 15$ m/s)
- Type of gear teeth
  - Internal gear teeth
  - External Gear teeth
  - Rack and pinion gear pair
- Position of gear teeth
  - Straight teeth
  - Inclined teeth
  - Curved Teeth

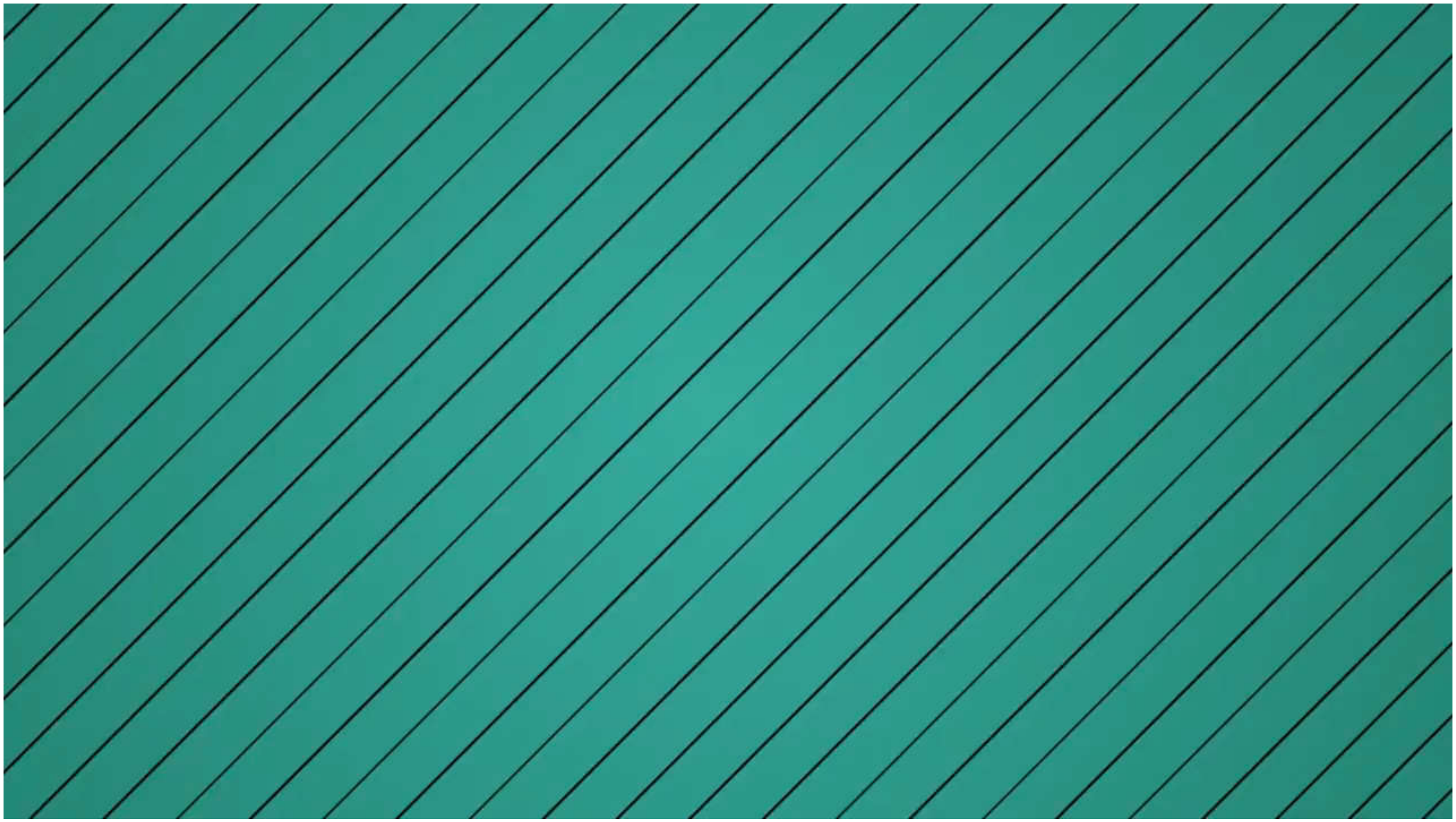
Classification animation of gears

Terminology of Gears



- Pitch Circle
- Pitch circle diameter
- Base circle
- Pitch Point
- Addendum
- Dedendum
- Face
- Flank
- Face width
- Circular pitch
- Diametral pitch
- Module
- Clearance
- Total Depth
- Working depth
- Tooth space
- Tooth Thickness
- Pressure line
- Pressure angle

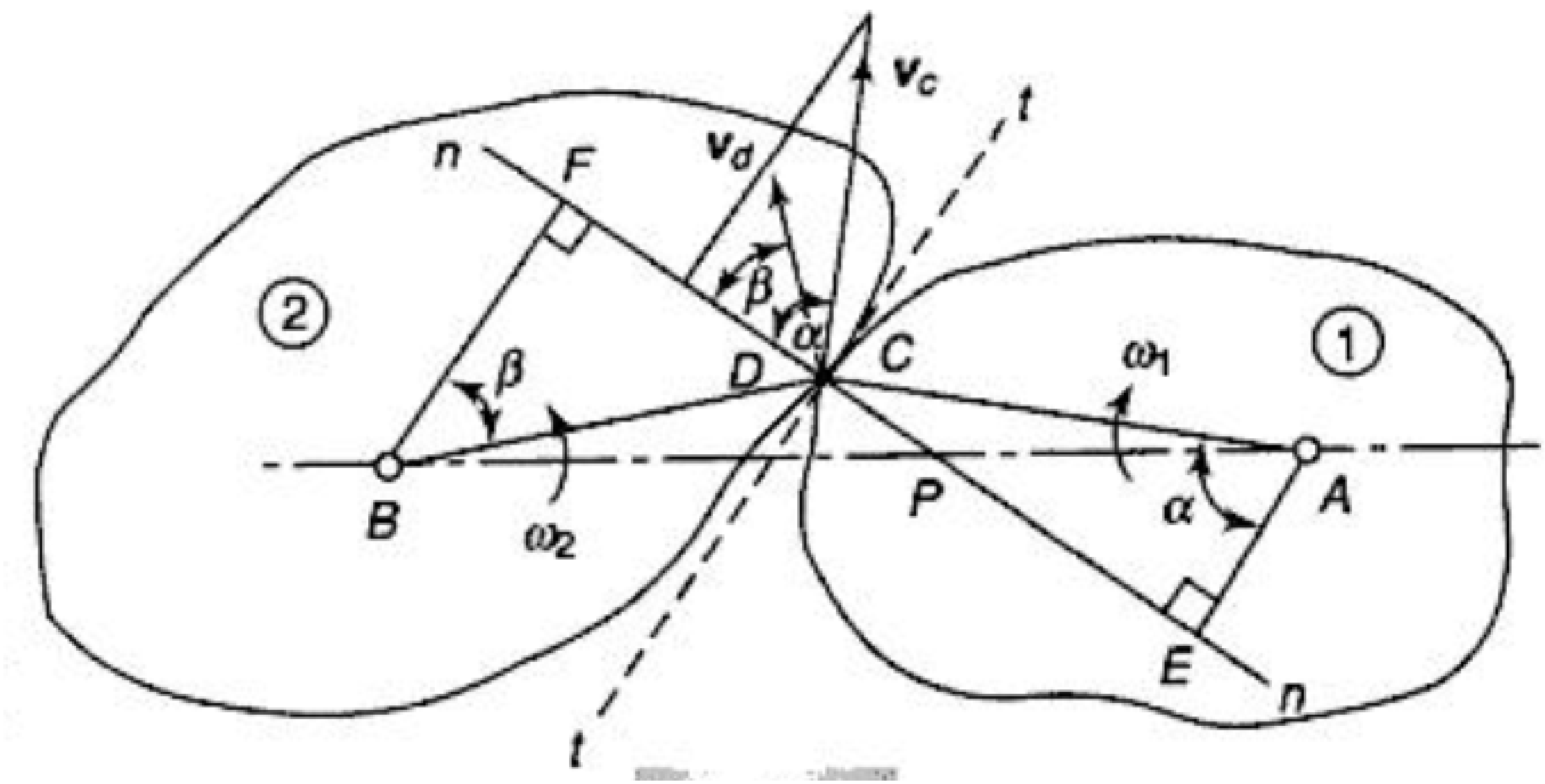
Terminology of gears animation



Condition for constant velocity ratio

- Aim:
  - To find condition for constant velocity ratio when gears are working.
- Before this let us understand few things first-
  - What is velocity ratio..?
  - Why do we need to maintain a constant velocity ratio during working?
- Constant Power ratio means – ratio of output to input
  - What does this indicate…?
  - This tells us how much power we are trying to transfer from one gear to another.
- But why we need to call it as constant velocity ratio..? Why not call it constant power ratio..?
  - Power = $\frac{2\pi NT}{60}$ = angular velocity × Torque
  - Since we cannot measure torque directly hence we consider it as constant and concentrate on the parameter which we can measure and control.

Condition for constant velocity ratio

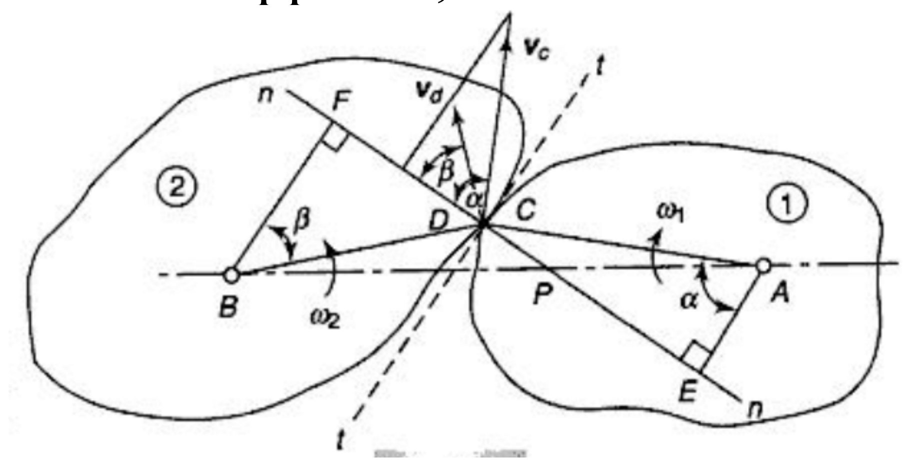


Condition for constant velocity ratio

- How any system will rotate if force is applied on it..?
  - If the applied force acts tangentially.
- Similarly the gears which are in contact rotate about each other due to the external force if it is applied tangentially to both the gears (i.e. common tangent)
- So whatever this external force is, it is divided into two components (Tangentially, Normally)
- What are the effects of these two component forces..?
  - Tangential component – it creates rotating action for the meshed gears
  - Normal component – it tries to separate the meshed gears.
- Hence in order to ensure that the gears to maintain contact, this total normal component should be zero.

Condition for constant velocity ratio

- Hence the total velocity component along common normal should be zero
- Velocity of gear 1 along the normal direction is $V_C \cos\alpha$
- Similarly for gear 2 velocity component is $V_D \cos\beta$
- The direction of these two velocity components are opposite, so the total velocity in the normal direction is
  - $V_C \cos\alpha - V_D \cos\beta = 0$
  - $V_C = \omega_1 \times AC$ and $V_D = \omega_2 \times BD$
  - *From* Δ *AEC,* $\cos\alpha = \frac{AE}{AC}$
  - *and* Δ *BFD,* $\cos\beta = \frac{FB}{BD}$*, substituting them*
  - $\omega_1 \times AE = \omega_2 \times FB$
  - *"P" is the pitch point and* Δ *AEP and* Δ*BFP are similar hence* $\frac{FB}{AE} = \frac{BP}{AP}$
  - *So substituting all these,* $\frac{\omega_1}{\omega_2} = \frac{FB}{AE} = \frac{BP}{AP} = \frac{D_2}{D_1} = \frac{T_2}{T_1}$

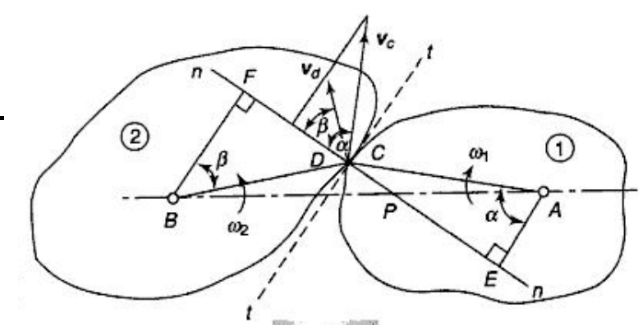


Condition for constant velocity ratio

- Physical significance of the expression:
  - The above expression gives us the ratio of angular velocity of gears which are in contact.
  - The ratio $\frac{BP}{AP}$ tells us that the total line AB is divided by point P.
- Hence combining the above two points –
  - The point P divides the center line AB in inverse ratio of their angular velocities.
  - So if we want to maintain a constant velocity ratio, the ratio of $\frac{BP}{AP}$ should be constant.
  - How to achieve the $\frac{BP}{AP}$ as constant..?
    - If the point P is fixed for all rotations of the gears, then this $\frac{BP}{AP}$ will be constant.
- **Law of gearing:**
  - **In order to have a constant velocity ratio, the common normal at the contact of two meshed gears should pass through the fixed pitch point.**

Velocity of Sliding

- It is the relative velocity of one tooth with respect to its meshed tooth along the common tangent at the point of contact.
- The velocity components along the tangential direction will cause sliding of gears
  - *V_C along tangential direction is* $\boldsymbol{V_C \sin \alpha}$*,* *$V_D$ along tangential direction is* $\boldsymbol{V_D \sin \beta}$
  - *Both these components acts in opposite direction, hence the total component of velocity in tangential direction is* $|\boldsymbol{V_C \sin \alpha - V_D \sin \beta}|$
  - $V_C = \omega_1 \times AC$ *and* $V_D = \omega_2 \times BD$
  - *From* $\Delta AEC, \sin \alpha = \frac{EC}{AC}$ *and* $\Delta BFD, \sin \beta = \frac{FD}{BD}$
  - *EC = EP + PC, FD = FP – PD and Point C = D*
  - *hence PD = PC*



- *Substituting all these in the expression,*
  - **Velocity of sliding =** $(\boldsymbol{\omega_1 + \omega_2})\ \boldsymbol{PC}$

Forms of tooth

- Form means Profile
- In actual practice there are two forms of teeth available:
  - Cycloid profile teeth
  - Involute profile teeth
- So now we need to learn about these teeth profiles, their pro's and con's, which is mostly used.
- Before going in to that let us learn the basic definition of the profiles
- Cycloid:
  - In geometry, a cycloid is the curve traced by a point on a circle as it rolls along a straight line without slipping.
- Involute
  - An involute of a curve is the locus of a point on a piece of taut string as the string is either unwrapped from or wrapped around the curve.

Animation of a Cycloid

Involute Animation

Differences between Cycloid and Involute

Cycloid	Involute
Pressure Angle is not constant	Pressure angle is constant
Two curves are required to design this profile	A Single curve is enough to obtain this profile
Manufacturing cost is high	Low manufacturing cost
Center distance between gears effect the velocity ratio	Center distance between gears do not effect the velocity ratio
No Interference	Interference occurs during working of gears.

Important points about Cycloid tooth

- The formation of cycloid tooth consists of both epicycloid and hypo cycloid.
- Hypocycloid is used as flank and epicycloid is used as face.
- In this tooth profile, length of approach = arc of approach
- Pressure angle is not constant and it varies from maximum at the beginning to zero at the contact point.
- The pitch point position varies if the center distance between the gears changes.

Important points of Involute tooth

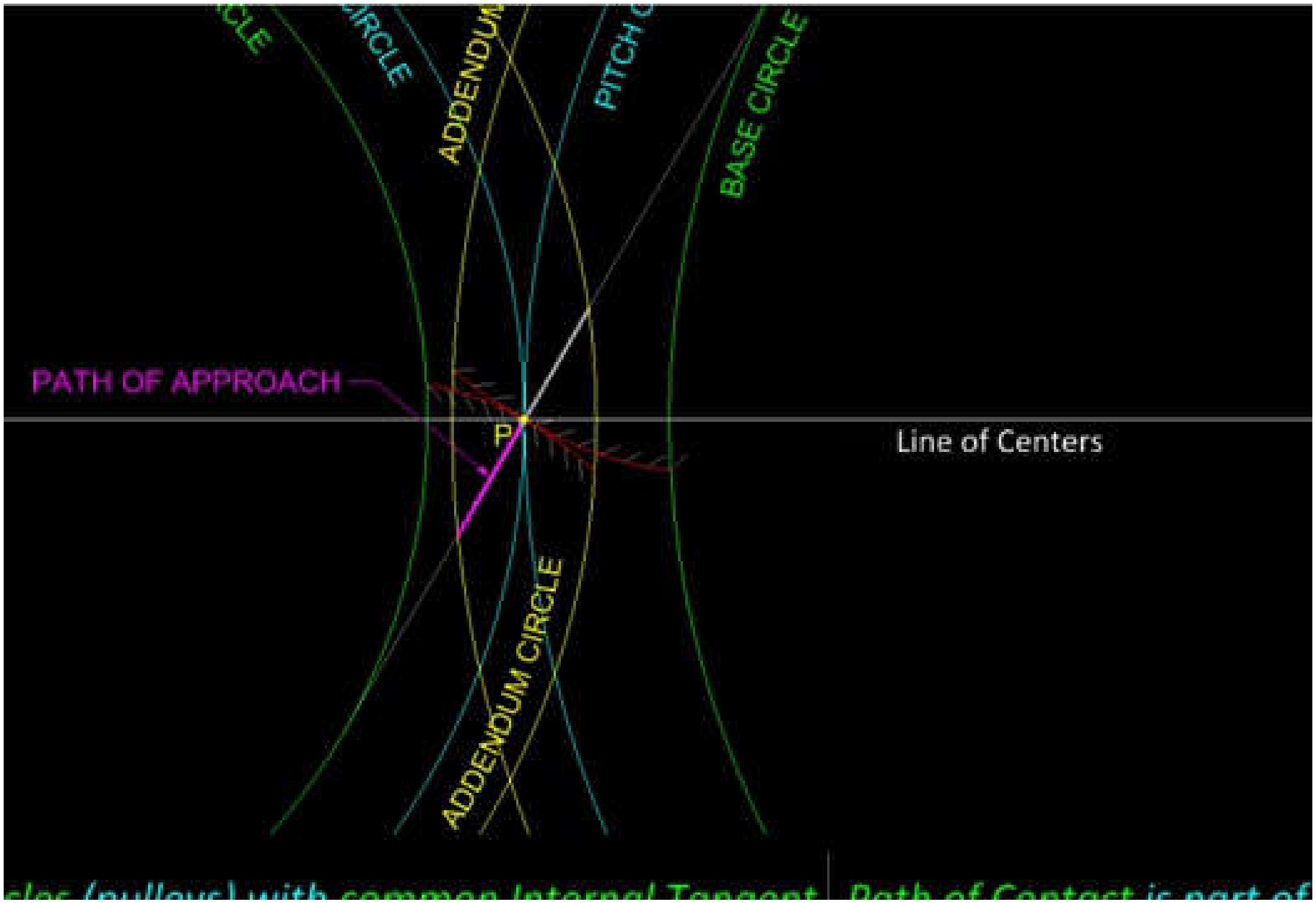
- Involute is the locus of point on a straight line when this straight line rolls over the circumference of a circle.
- The profile of involute tooth is made of a single curve.
- The line of action of involute tooth is along common normal at the contact point.
- Pressure angle remains constant.
- The pitch point position is fixed and will not change even if the center distance is changed.

Length of contact

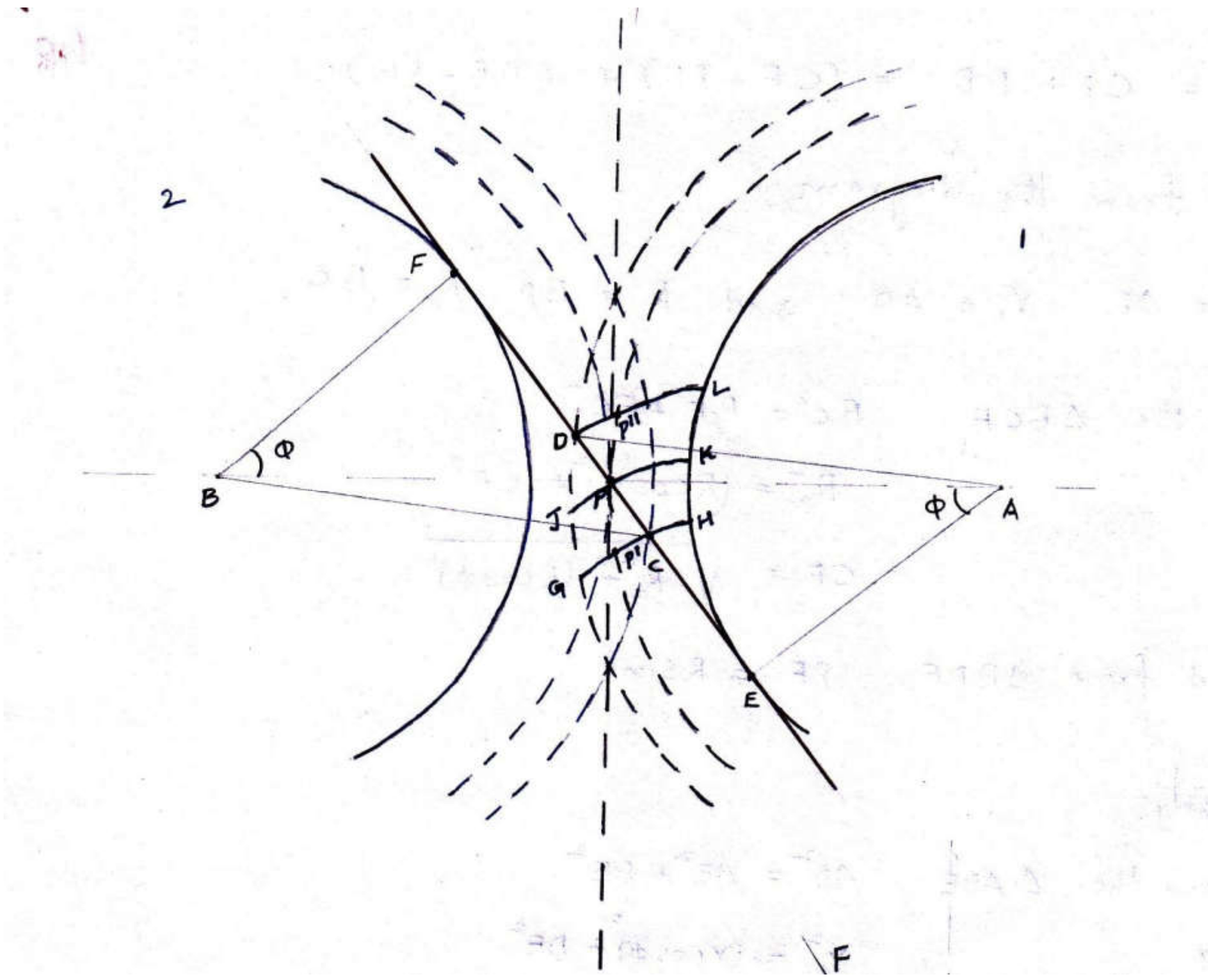
- It is defined as locus of point of contact of two mating teeth along a straight line from beginning of contact to end of contact
- This is also known as path of contact which is sum of path of approach and path of recess.

Length of contact

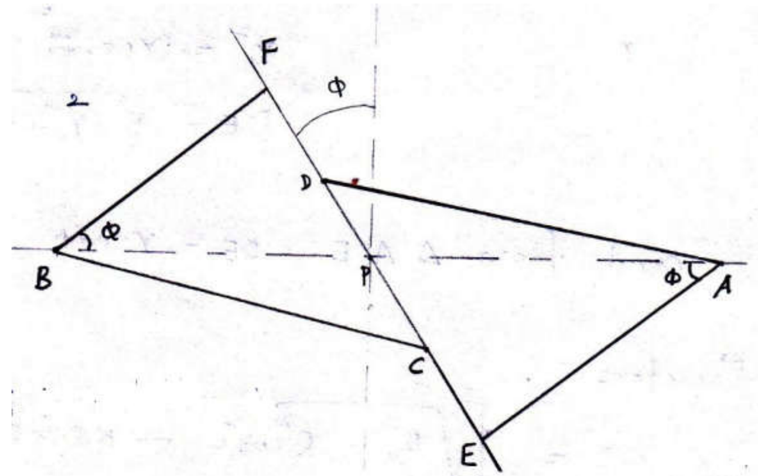
Length of contact



Length of contact



Length of contact



- Point A is center for gear 1 and B is center for gear 2.
- Let gear 1 be the small driving gear, and AP is pitch circle radius. Similarly gear 2 is driven gear and BP is its pitch circle radius.
- Path of contact (CD) = Path of approach (PD) + Path of Recess (PC)
- From the diagram CD = (DE – PE) + (CF – PF)
- Let us consider: r = AP, r_a = AD and R = BP, R_a = BC
- EF is common tangent for base circles of both gears
- From *ΔBCF,*
  - $BC^2 = BF^2 + FC^2$
  - $R_a^2 = (R\cos\phi)^2 + CF^2$
  - $CF = \sqrt{(R_a^2 - (R\cos\phi)^2)}$
- From ΔBPF, PF = R sin ϕ
- From ΔADE
  - $AD^2 = AE^2 + DE^2$
  - $r_a^2 = (r\cos\phi)^2 + DE^2$
  - $DE = \sqrt{(r_a^2 - (r\cos\phi)^2)}$
- From ΔAPE, PE = r sin ϕ

Length of contact

- Substituting the above obtained values,
- Length of contact
- CD = $\left[\sqrt{R_a^2 - (R\cos\phi)^2} - \mathrm{R}\sin\phi\right] + \left[\sqrt{r_a^2 - (r\cos\phi)^2} - \mathrm{r}\sin\phi\right]$
- CD = $\sqrt{R_a^2 - (R\cos\phi)^2} + \sqrt{r_a^2 - (r\cos\phi)^2} - (\mathrm{R} + \mathrm{r})\sin\phi$

  - **Velocity of sliding during engagement =** $(\boldsymbol{\omega_1} + \boldsymbol{\omega_2})\ \boldsymbol{PC}$
  - **Velocity of sliding during dis engagement =** $(\boldsymbol{\omega_1} + \boldsymbol{\omega_2})\ \boldsymbol{PD}$

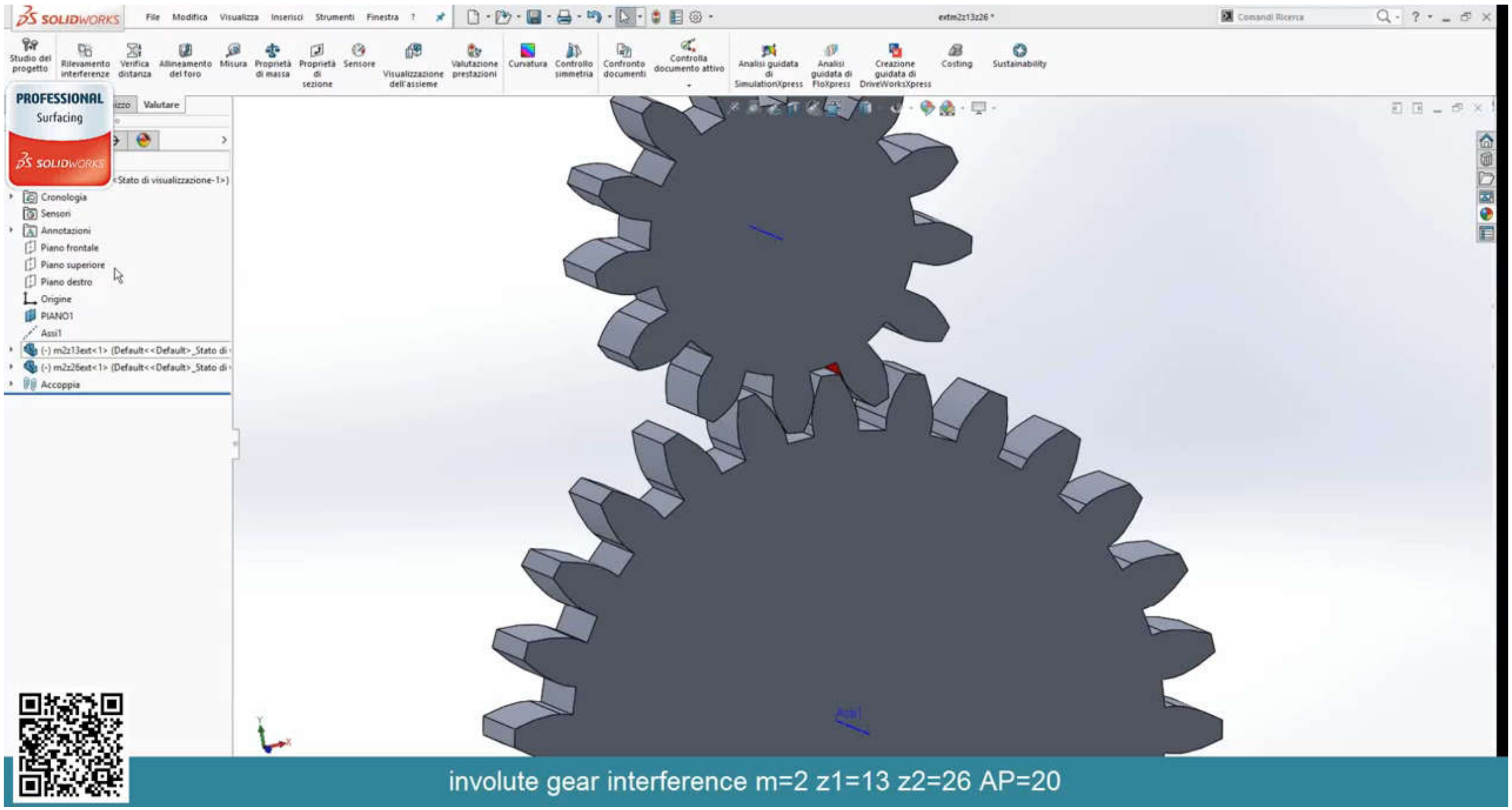
Arc of contact

- It is defined as locus of point of contact of two mating teeth on pitch circle of gear/pinion from beginning of contact to end of contact.
- Arc of contact = arc of approach + arc of recess
- Arc of contact = $\frac{length\ of\ contact}{\cos \emptyset}$
- Number of teeth in contact during an instant = $\frac{arc\ of\ contact}{circular\ pitch}$

Interference

- During meshing of two gears, proper power transmission will happen if the tooth profiles are similar (i.e. conjugate teeth profile)
- But if due to some reason one gear teeth got damaged to its profile, then during meshing of gears proper power transmission cannot happen. Such gears are called as non conjugate gears.
- So when two non conjugate gears are meshed then their surface will not have a common tangent. This phenomenon is known as interference.
- Meshing of two non conjugate gears creates a phenomenon known as interference.

Interference



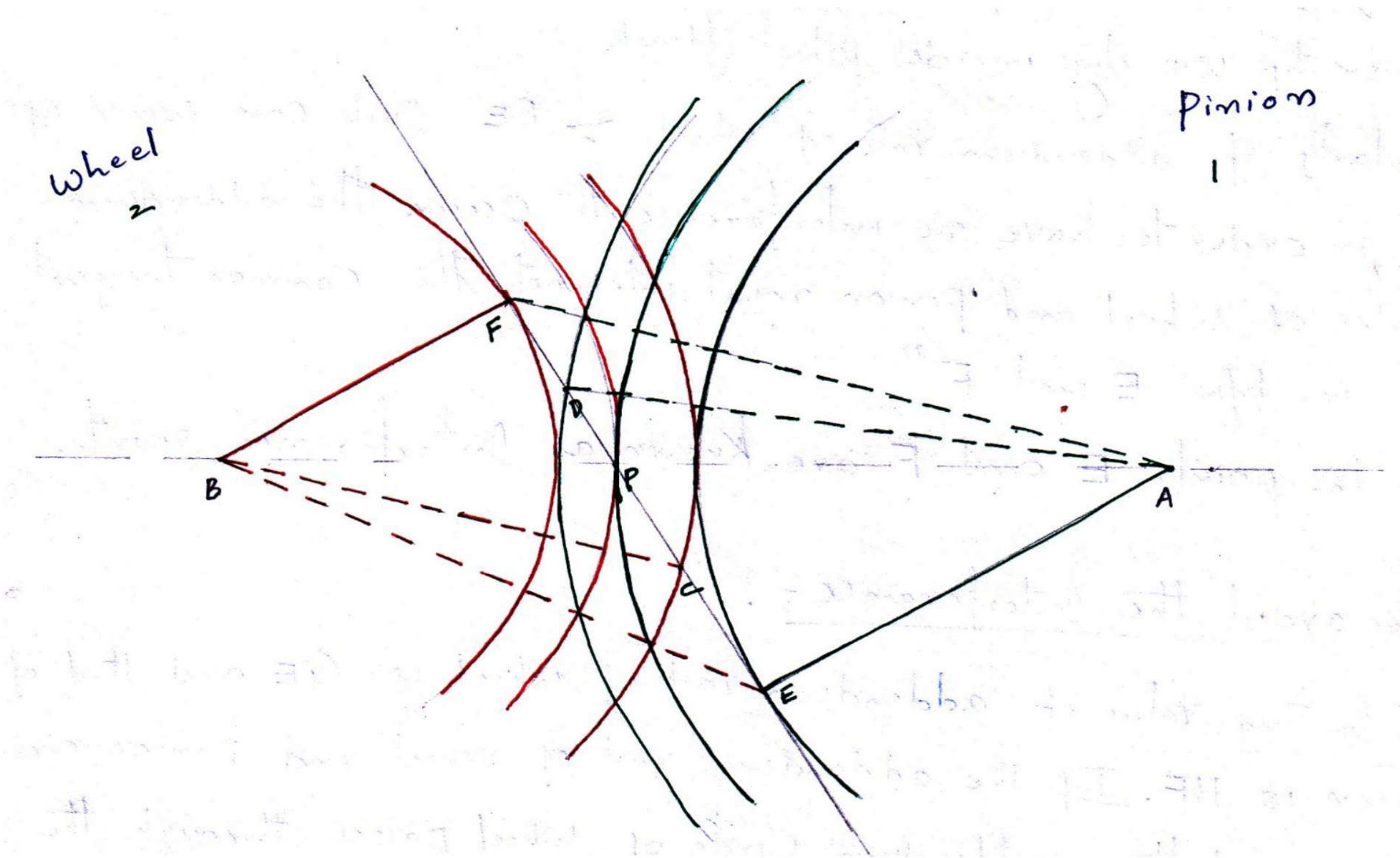
Interference

- How to avoid something from happening?
  - By knowing how it is actually happening
- So now to avoid interference to happen we should know how it is happened.
- So let us find the cause for happening of interference..
- What do we know from the definition..?
  - Interference is phenomenon occurs when non conjugate teeth are meshed.
- What will happen when non conjugate teeth are meshed?
  - Do we have any power transfer interruption ?
    - No
  - So what damage it is going to do..?
    - Common tangent of the base circles will be disturbed.

Interference

- If common tangent is disturbed, then what will be the problem?
  - Proper gear ratio cannot be maintained.
  - During a long run, the surface of the gears will be damaged due to crushing action between the teeth.
- So the cause is found for occurrence of interference…?
  - Common tangent if not maintained during meshing of gears then interference will be occurred.
- Again we will have another question
  - How can we say if common tangent is maintained or not during meshing of gears?
  - The answer for this question will be understood at last.

Interference



Interference

- The controlling parameter whether interference to occur or not is addendum circle radius.
- From the fig. it is evident that the teeth will engage at C and disengage at D
- Now if we try to increase the radius of addendum of pinion, the maximum radius we can increase is till point F (i.e. AF will be the maximum and new radius of addendum circle of pinion)
- The similar situation exists for gear wheel, for which BE will be the maximum and new radius of addendum circle.
- If there is any further increase in radius of addendum circle, then what will happen..?
  - Common tangent cannot be maintained.
  - Crushing of gear teeth will occur.

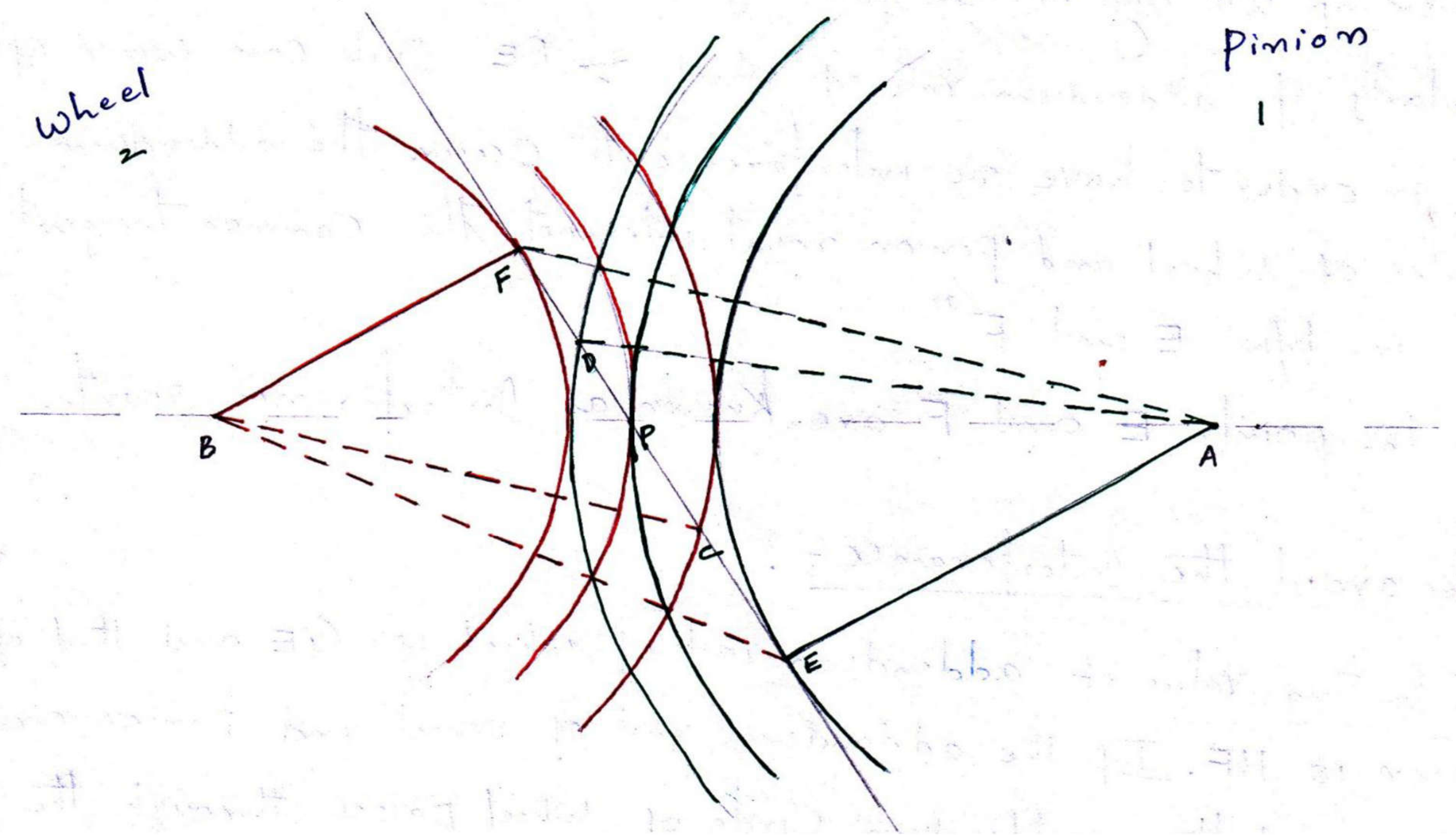
Interference

- So now getting back to what we want to learn..?
  - Aim:
    - To know the root cause for creating the interference phenomenon
  - From our previous discussion it is understood that if common tangent is not maintained then interference occurs.
- Result from our discussion:
  - Common tangent cannot be maintained if the addendum circle radius of pinion/wheel exceeds their respective maximum values (i.e. AF/BE)
- Hence as per our topic we need to avoid this interference phenomenon.
- How to avoid..?
  - By not exceeding the addendum circle radius beyond its maximum.

How to avoid interference…?

- By not exceeding the addendum circle radius beyond its maximum.
- How to do this…?
- If we decrease the addendum circle radius then the whole gear will become smaller, which means that the number of teeth will also decreases.
- Hence the way to avoid the interference is to control the addendum radius, but during this as the teeth will also be reduced we need to find the least number of teeth that will be present on the gear.
- So we need to find the minimum number of teeth on the gear so that beyond that number we should not reduce the teeth since it will effect the working of gear.

Interference

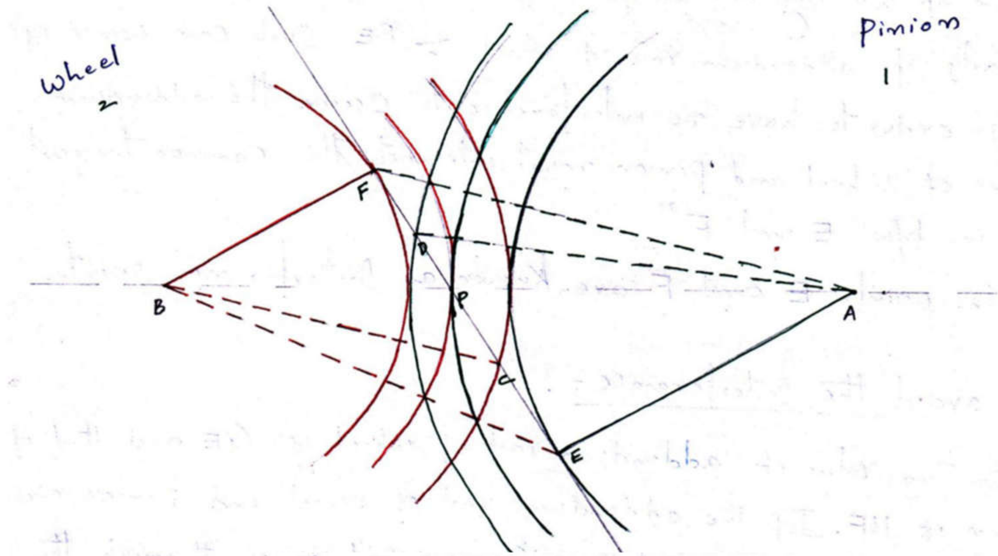


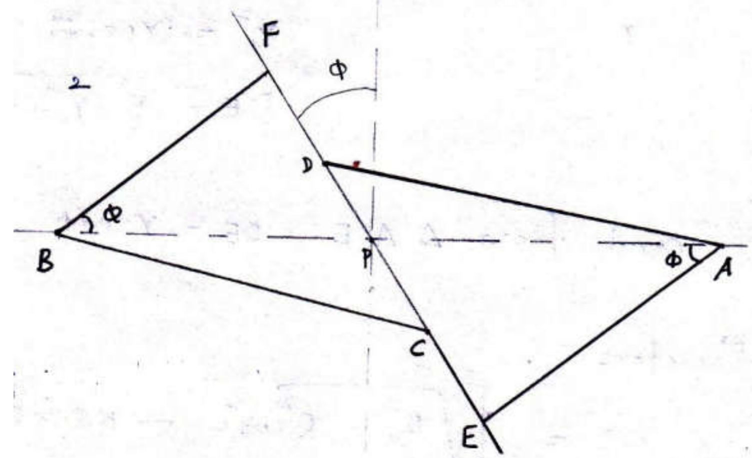
Minimum number of teeth to avoid interference.

- Aim-
  - To find the minimum number of teeth to avoid interference.
- Condition-
  - To avoid interference, as per the diagram, the addendum circle radius of wheel should be less than BE, for pinion it should be less than AF.
- Considerations-
  - Let R be the Pitch circle radius of wheel, so to avoid interference
  - Original addendum ≤ maximum addendum
  - BE - R = Maximum addendum
  - BE – R means addendum of the gear (which is the maximum value).
  - So let us find this addendum value and apply a limiting condition to get our required aim (minimum number of teeth).

Minimum number of teeth to avoid interference.

- Finding Addendum of the wheel-
- We know that max addendum of wheel = BE – R
- So first let us find the value of BE
  - *Consider the ΔBFE from the diagram*
  - $BE^2 = FB^2 + FE^2 = FB^2 + (FP+PE)^2$
  - $BE^2 = (R\cos\varphi)^2 + (R\sin\varphi + r\sin\varphi)^2$
  - $BE^2 = R^2 + (r^2 + 2R.r)(\sin\varphi)^2$
  - $BE^2 = R^2\left[1 + \frac{r}{R}\left[\frac{r}{R} + 2\right](\sin\varphi)^2\right]$
  - $BE = \sqrt{R^2\left[1 + \frac{r}{R}\left[\frac{r}{R} + 2\right](\sin\varphi)^2\right]}$
- *Hence max addendum of wheel* $= \sqrt{R^2\left[1 + \frac{r}{R}\left[\frac{r}{R} + 2\right](\sin\varphi)^2\right]} - R$





Minimum number of teeth to avoid interference.

- But, to find the minimum number of teeth on wheel, we need to use the limiting condition.
- Limiting condition-
  - Maximum addendum of wheel ≥ original addendum of the wheel.
- Original addendum of wheel ≤ R $\left[\sqrt{\left[1+\frac{r}{R}\left[\frac{r}{R}+2\right](\sin\varphi)^2\right]}-1\right]$
  - Manipulating the above equation for our convenience,
  - $\frac{r}{R}=\frac{t}{T}$ and R = $\frac{m.T}{2}$ and G = $\frac{T}{t}$
  - Hence max addendum = $\frac{m.T}{2}\left[\sqrt{\left[1+\frac{t}{T}\left[\frac{t}{T}+2\right](\sin\varphi)^2\right]}-1\right]$
- Original addendum = m.a_w {m – module and a_w – addenda}
- So equating both we get T ≥ $\frac{2.a_w}{\left[\sqrt{\left[1+\frac{1}{G}\left[\frac{1}{G}+2\right](\sin\varphi)^2\right]}-1\right]}$

Important Formulae

- Length of Path of Contact: $\left[\sqrt{R_a^2 - (R\cos\phi)^2} - \mathrm{R}\sin\phi\right] + \left[\sqrt{r_a^2 - (r\cos\phi)^2} - \mathrm{r}\sin\phi\right]$
- Length of arc of contact = $\dfrac{Length\ of\ path\ of\ contact}{\cos\phi}$
- Velocity of sliding during engagement = $(\omega_1 + \omega_2)\ PC$
- Velocity of sliding during disengagement = $(\omega_1 + \omega_2)\ PD$
- Minimum number of teeth on wheel to avoid interference T = $\dfrac{2.a_w}{\left[\sqrt{\left[1+\frac{1}{G}\left[\frac{1}{G}+2\right](\sin\varphi)^2\right]}-1\right]}$
- Minimum number of teeth on pinion to avoid interference t = $\dfrac{2.a_p}{\left[\sqrt{\left[1+G(G+2)(\sin\varphi)^2\right]}-1\right]}$