UNIT-1

GYROSOPIC COUPLE AND PRECESIONAL MOTION

Course Objective: To help students to understand the gyroscopic effect on vehicles, ships and planes.

Course Outcome: The student will be able to apply the knowledge of gyroscopic couple

Syllabus

- Introduction
- Precessional and angular motion
- Gyroscopic couple
- Effect of gyroscopic couple on
  - Aero plane
  - Naval Ship
- Stability of automobiles
  - Four wheel automobile while moving in a curved path

Introduction

- Title of the chapter contains two individual topics:
  - Gyroscopic couple
  - Precessional motion.
- What is a couple…?
  - A turning force which tends a body to move in a curved path w.r.t an axis.
- So Gyroscopic couple means..?
  - Gyroscopic turning force making the body to rotate about an axis…
- What is this Gyroscopic Turning force and what type of effects it is going to create is our discussion of interest.
- Next topic in our title is --- Precessional Motion..
- What is precessional motion..?
  - Making a body/system to move in a particular path without any failure.

Gyroscopic Couple

- Let us learn this topic using the 3W Concept → What, When, Where
  - When it is going to occur in the system…?
  - Where is it going to act on the system…?
  - What is a gyroscopic couple…?
- When it is going to occur in the system..?
  - When a system is experiencing 2 simultaneous rotational motions whose axis are perpendicular to each other, then this gyroscopic force will be created.
- Where is it going to act on the system..?
  - On the axis perpendicular to both the axis.
- What is a gyroscopic couple..?
  - It is a rotating force acting on a system when the system is experiencing with 2 simultaneous rotational motions whose axis are perpendicular to each other, then on the third perpendicular axis this force will be created.

Active Force and Reactive Force

- Active force → gyroscopic force created by the system due to its simultaneous dual rotations about perpendicular axis
- Reactive force → opposite gyroscopic force created manually in order to avoid the effect caused by the active gyroscopic force.
- Why apply a reactive force..?
  - To keep the system in a stable equilibrium position.

Terminology

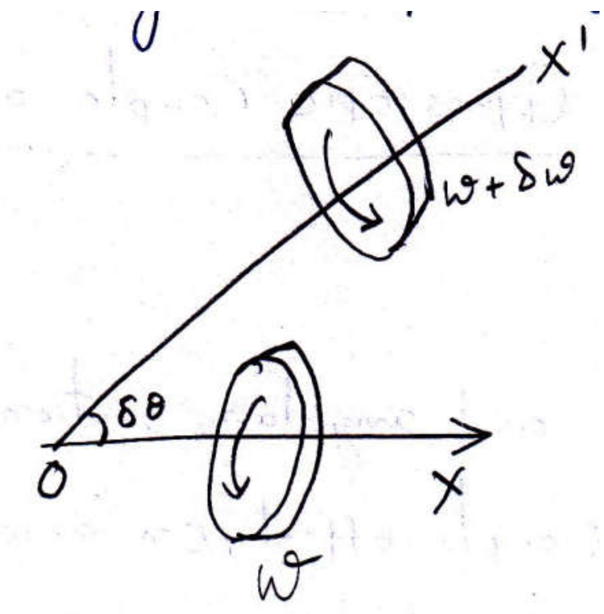
- Let us discuss the various terminology used in this by considering a situation:
  - Let a disc is mounted on a shaft is rotating with a certain angular velocity (ω) about X-axis, now the shaft is made to rotate about Z-axis with angular velocity (ω_p)
- So in the above situation we have 2 simultaneous rotational motion occurring in the system and the axis of these motions are perpendicular.
  - Hence our active gyroscopic couple will be created on an axis perpendicular to both of these two axis (i.e. Y-axis)
- Terminology:
  - X- axis : Axis of spin
  - Z- axis : Axis of precession
  - Y- axis : Axis of active or reactive gyroscopic couple

Magnitude of Gyroscopic Couple

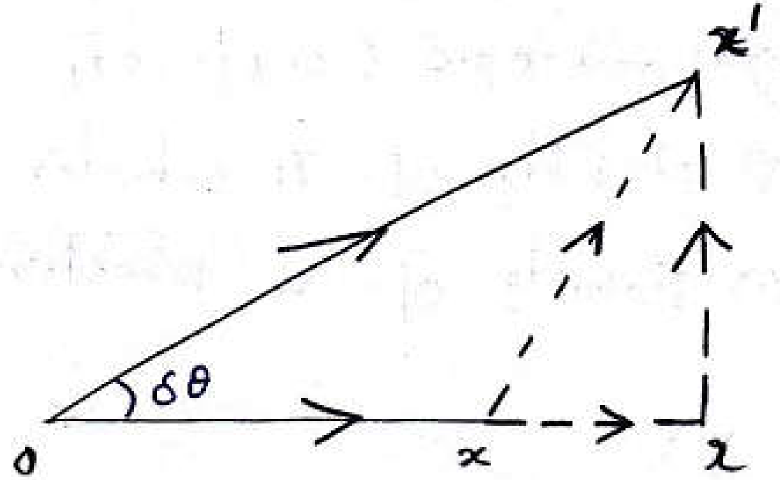
- Aim:
  - To make the system in a balanced position.
- How:
  - By knowing the magnitude of the active gyroscopic couple.
- New Aim:
  - Find the magnitude of active gyroscopic couple.
- What is the formula for Force (as per newton's 2^{nd} law)
  - $F = m \times a$ [this formula is applicable if system motion is linear]
  - So what happens when system motion change from linear to rotating…?
    - Force becomes couple
- Now what happens if force becomes couple
  - Mass becomes moment of inertia (I) and linear acceleration becomes angular acceleration (α)
- Now the formula for Couple is
  - $C_G = I \times \alpha$
  - And since we need gyroscopic couple, the α will be known as angular acceleration of the system moving in 3^{rd} perpendicular direction/axis (i.e. Y-axis)

Magnitude of Gyroscopic couple

- Let us consider the same situation and find the Magnitude of gyroscopic couple:
    - Situation: Disc on shaft spinning with ω rad/sec (about X-axis). Now shaft is made rotated about Z-axis with a angular displacement of δθ.
- Let I be the moment of inertia of the disc.
- Hence as per our vector theory xx^I represents the total change in angular velocity
- Total angular acceleration = $\alpha_r + \alpha_N$
- Since we need the perpendicular component only, so we require α_N

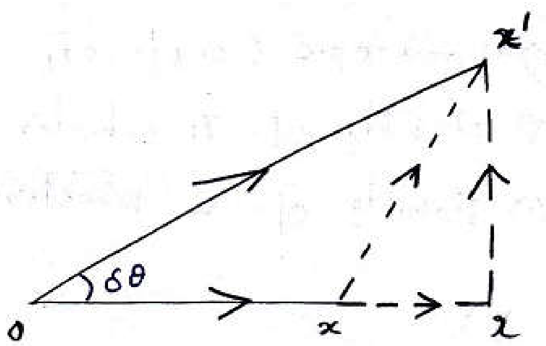


Magnitude of Gyroscopic couple



- Let us draw the velocity diagram for the above situ
- From the velocity diagram,
  - o*x – initial angular velocity = ω*
  - *ox^I – final angular velocity = ω + δω*
  - *xx^I – Change in angular velocity*
- In order to change its position, from *x* to *x^I* the system should travel in both horizontal and vertical path (i.e. radial and normal)
  - *xr – radial direction*
  - *x^Ir – Normal direction*
- As per our previous discussion, from Newton 2nd law, the formula for gyroscopic couple is $C_G = I \times \alpha_N$

Magnitude of Gyroscopic couple



- Normal acceleration $\alpha_N = \frac{x^I r}{\delta t}$
- So from the diagram $\Delta o r x^I$ → $sin\ \delta\theta = \frac{x^I r}{o x^I}$ → $x^I r = o x^I\ sin\ \delta\theta$
- Hence $\alpha_N = \frac{o x^I\ sin\ \delta\theta}{\delta t} = \frac{(\omega + \delta\omega)\ sin\ \delta\theta}{\delta t} = \frac{\omega\ sin\ \delta\theta + \delta\omega\ sin\ \delta\theta}{\delta t}$
- *Assuming δθ is very small and hence sin δθ = δθ and since δθ itself is small so multiplication of δω with δθ will be very small so we can ignored.*
- Therefore $\alpha_N = \frac{\omega\ \delta\theta}{\delta t} = \omega\ .\ \frac{\delta\theta}{\delta t}$
- **Hence** $\mathbf{C_G = I.\ \alpha_N = I\ \omega\ .\ \frac{\delta\theta}{\delta t}}$
- $\left[\frac{\delta\theta}{\delta t}\ is\ change\ of\ angular\ displacement\ w.r.t\ time\ about\ Z - axis\ (precesion\ axis)\right]$
- So $\frac{\delta\theta}{\delta t} = \omega_p$ → hence $C_G = I\ \omega\ \omega_p$

Effects of Gyroscopic couple on Aero plane

- The front portion of aero plane consists of a rotor/propeller which rotates with a certain rpm, it is used to move the plane in forward direction.
- There is a tail attached at the back side of plane, through which the direction of the plane can be controlled.
- Situation: Turning the aero plane
- Aim: To analyse what happens if the aero plane turns.

Gyroscopic effects on aero plane (contd.)

ω = Angular velocity of the engine in rad/s,

m = Mass of the engine and the propeller in kg,

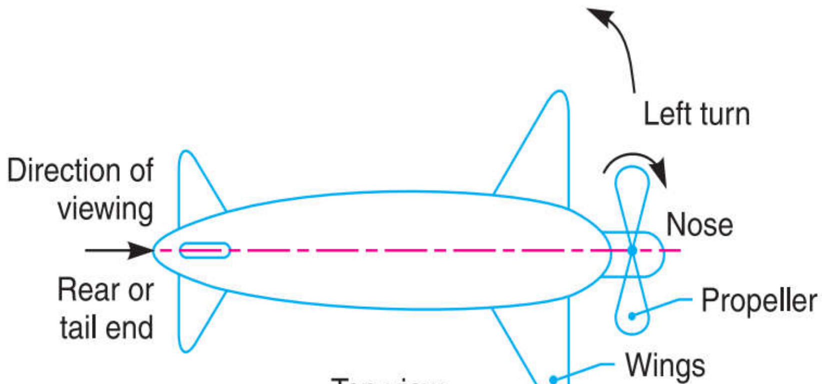
k = Its radius of gyration in metres,

I = Mass moment of inertia of the engine and the propeller in kg-m^2 $= m.k^2$,

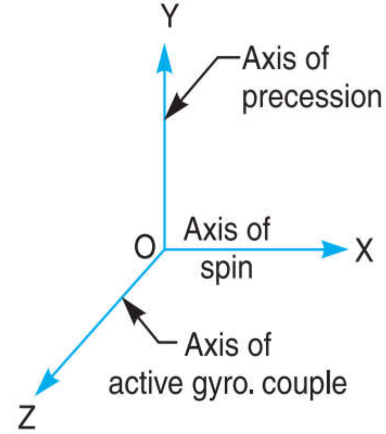
v = Linear velocity of the aeroplane in m/s,

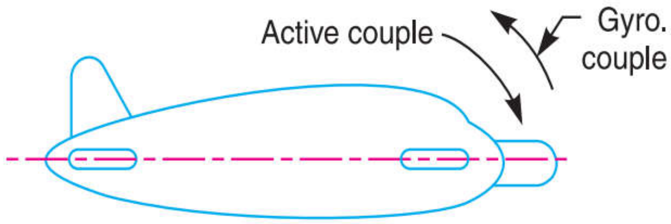
R = Radius of curvature in metres, and

$$\omega_P = \text{Angular velocity of precession} = \frac{v}{R} \text{ rad/s}$$

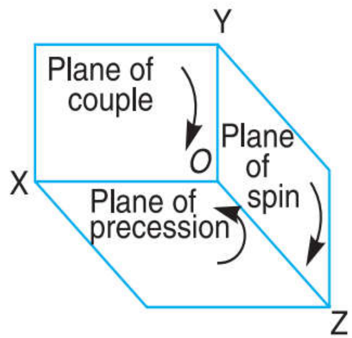


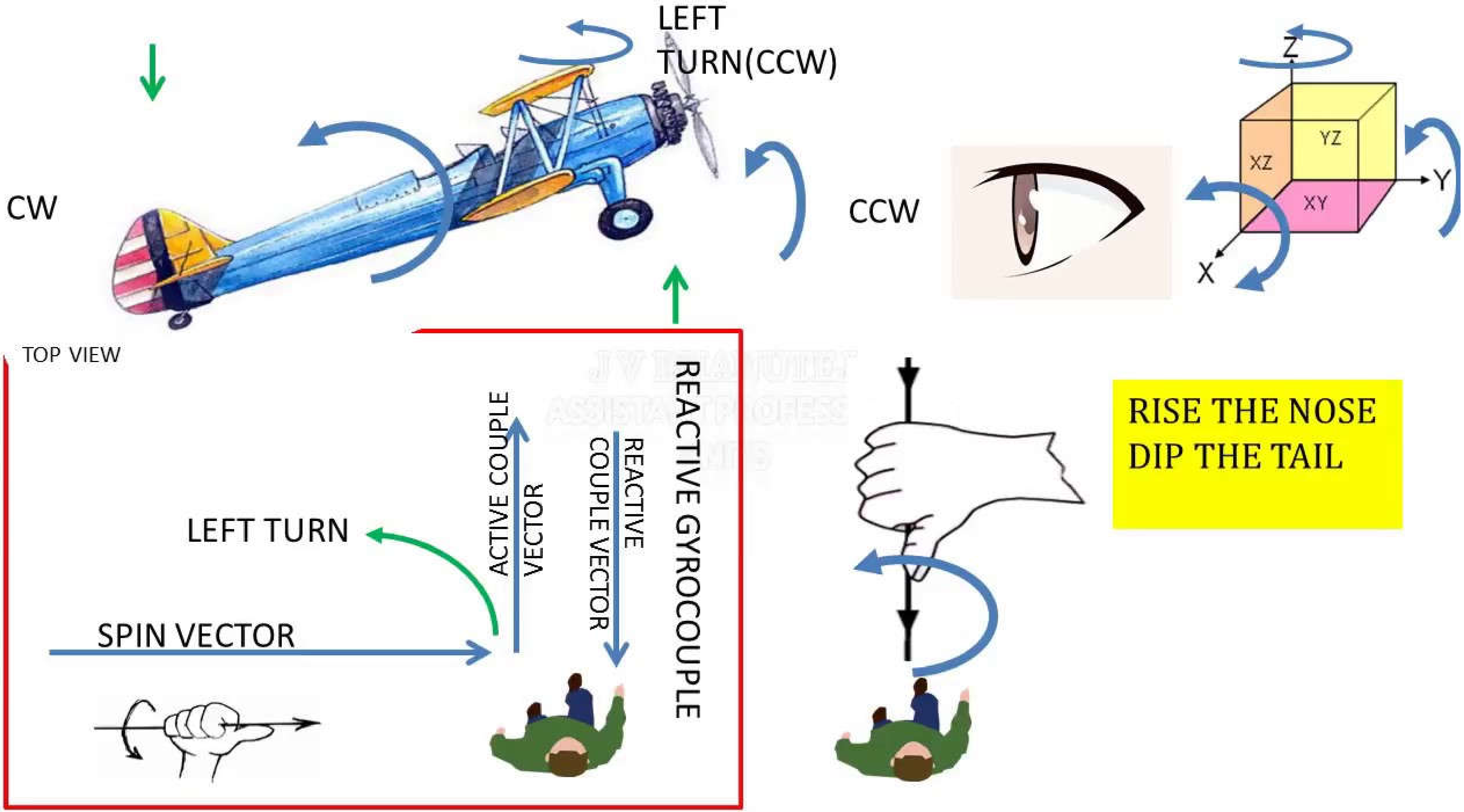
Top view





Front view




LEFT
TURN(CCW)
CW
CCW
Z
Y
X
YZ
XZ
XY
TOP VIEW
ACTIVE COUPLE VECTOR
REACTIVE COUPLE VECTOR
REACTIVE GYROCOUPLE
LEFT TURN
SPIN VECTOR
RISE THE NOSE
DIP THE TAIL

Gyroscopic effects on aero plane (contd.)

Looking from Rear End (Back Side)

Direction of Rotation of propeller	Direction of Motion	Active G.C	Reactive G.C
Clockwise	Turning Left	Lower the nose, Raise the tail	Raise the nose, Lower the tail
Clockwise	Turning Right	Raise the nose, Lower the tail	Lower the nose, Raise the tail
Anti-Clockwise	Turning Left	Raise the nose, Lower the tail	Lower the nose, Raise the tail
Anti-Clockwise	Turning Right	Lower the nose, Raise the tail	Raise the nose, Lower the tail

Effect of Gyroscopic couple on Naval Ship

- Terminology of a ship

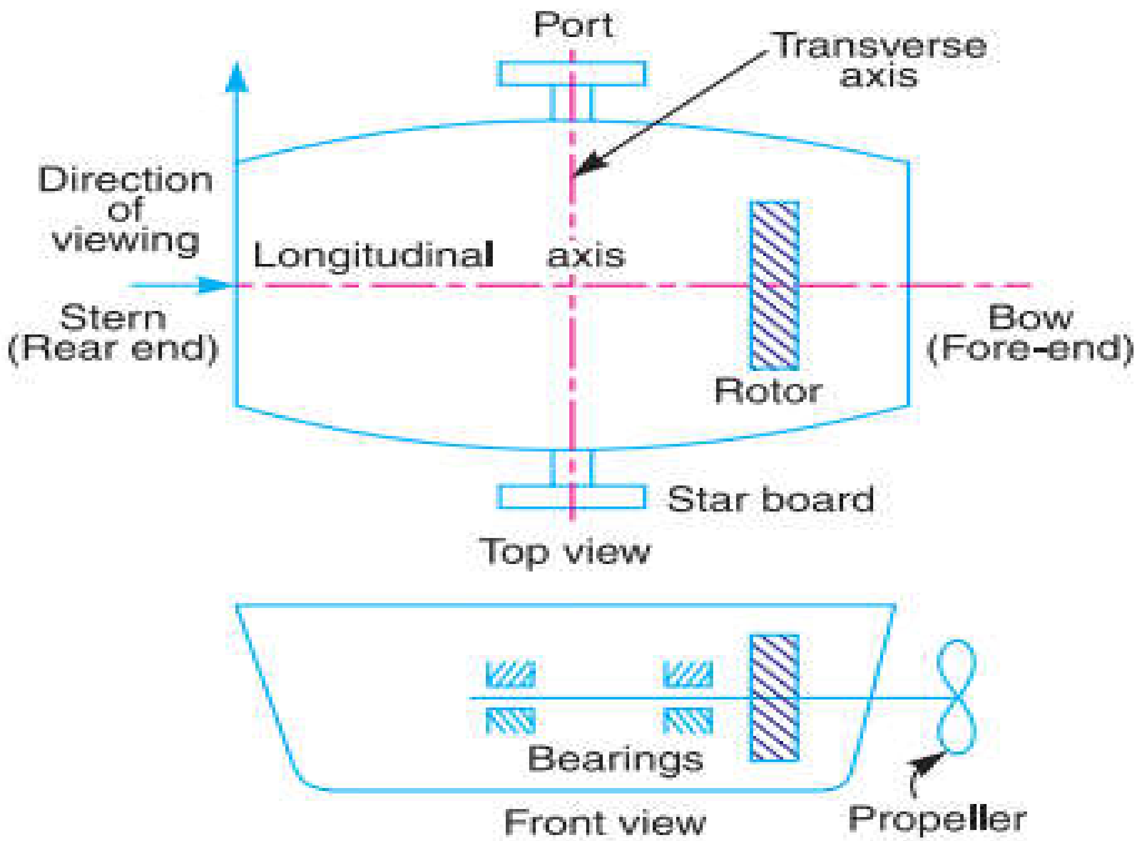


Fig. 14.7. Terms used in a naval ship.

Effect of Gyroscopic couple on Naval Ship

- In case of ship the gyroscopic effect is discussed in 3 different cases:
  - Steering
    - Turning the ship in right/left direction while it is moving forward.
  - Pitching
    - Movement of a ship in up and down direction in a vertical plane about the transverse axis (Y-axis).
  - Rolling
    - Rotating of the ship along the axis.

Steering Action:

- Steering action of the ship is similar to that of the of aero plane which is turning in right/left direction.
- So the effect of gyroscopic and the magnitude also remains same.

Effect of Gyroscopic couple on Naval Ship (steering)

Looking from Rear End (Back Side)

Direction of Rotation of rotor	Direction of Motion	Active G.C	Reactive G.C
Clockwise	Turning Left	Lower the Bow, Raise the Stern	Raise the Bow, Lower the Stern
Clockwise	Turning Right	Raise the Bow, Lower the Stern	Lower the Bow, Raise the Stern
Anti-Clockwise	Turning Left	Raise the Bow, Lower the Stern	Lower the Bow, Raise the Stern
Anti-Clockwise	Turning Right	Lower the Bow, Raise the Stern	Raise the Bow, Lower the Stern

Effect of Gyroscopic couple on Naval Ship (Pitching)

Pitching Action:

- Movement of a ship in up and down direction in a vertical plane about the transverse axis.

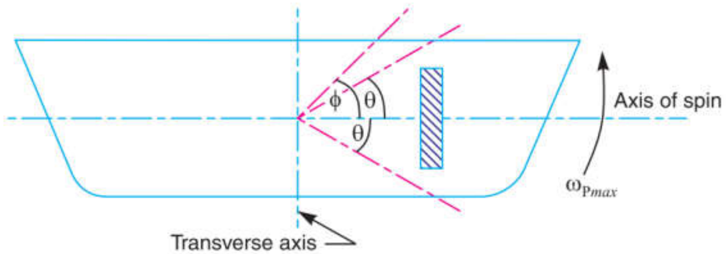
Effect of Gyroscopic couple on Naval Ship (Pitching)

- In this action the ship pitches about the transverse axis (i.e. Z-axis)
- Assumption: for the purpose of analysis we assume that the Ship pitches in form of S.H.M.
- Effect of Gyroscopic couple on ship during pitching:
  - Use Right hand thumb rule
  - Look from Top/Bottom View.

Rotor of the Ship is rotating in Anti - Clockwise direction when viewing from rear end		
Direction of Pitching	**Active G.C**	**Reactive G.C**
Upwards	Couple applied on Port Side	Couple to be applied on star board side
Downwards	Couple applied on Star-board Side	Couple to be applied on port side.

Magnitude of gyroscopic couple during Pitching

- The general formula for gyroscopic couple is –
  - $C_G = I.\ \omega.\ \omega_p$
- Where as in case of pitching we assumed the precession motion of ship as SHM.
- Hence as per SHM, the angular displacement equation is
  - $\theta = \Phi \sin \omega_1 t$
  - ω_1 represents natural frequency = $\frac{2\pi}{t_p}$
  - And angular velocity of precesion $(\omega_p) = \frac{d\theta}{dt} = \Phi.\ \omega_1.\ \cos \omega_1 t$
  - The maximum value for cos is 1, so using this we get, $(\omega_p)_{max} = \Phi.\ \omega_1$
  - Where Φ represents the angular displacement (amplitude of swing from mean position)



  - Hence $(C_G)_{max} = I.\ \omega.\ (\Phi.\ \omega_1)$

Effect of Gyroscopic couple on Naval Ship (Rolling)

- Basic Rule for Gyroscopic force to act on the system:
  - Two simultaneous rotations should happen and their respective axis should be perpendicular
  - Then only gyroscopic couple will act on the system on the 3rd perpendicular axis.
- In case of rolling the axis of spin and axis of precession are parallel, hence as both these axis are not perpendicular, No Gyroscopic effect is observed.

Stability of a System (Automobile)

- Stability means:
  - The property of a body to move in a steady motion without effecting from any of the external forces.
- In case of automobiles we have basically 2 types:
  - Four wheel automobile
  - Two wheel automobiles.
- So what are the forces acting on an automobile while it is moving in SL Motion and trying to take a turn:
  - Weight of the automobile
  - Force due to centrifugal couple
  - Force due to Gyroscopic Couple.

Analysis of Four Wheel automobile

- Aim:
  - To maintain stability of the automobile.
- As discussed, three forces will act on the system which is in motion and taking a turn.
- Where does these forces act on our automobile..?
  - On the wheels of the automobile.
- So we need to find the magnitude of the **resultant force** acting on the wheels of the automobile.
- So let us discuss about each force acting on the automobile system in detail.

Analysis of Four wheel automobile

- **Situation:**
- We have an automobile with four wheels moving in forward direction taking a turn (say left turn).
- Let us take the following parameters:
  - *m – mass of the vehicle in kg*
  - r_w *– radius of the wheels*
  - *R – radius by which the vehicle turns*
  - I_w *– moment of inertia of the wheels*
  - I_E *– moment of inertia of rotating components of engine*
  - ω_w *– angular velocity of the wheels*
  - ω_E *– angular velocity of the rotating engine components.*
  - *G – gear ratio* $= \frac{\omega_E}{\omega_w}$
  - *v – linear velocity of the automobile* $= r_w \, \omega_w$

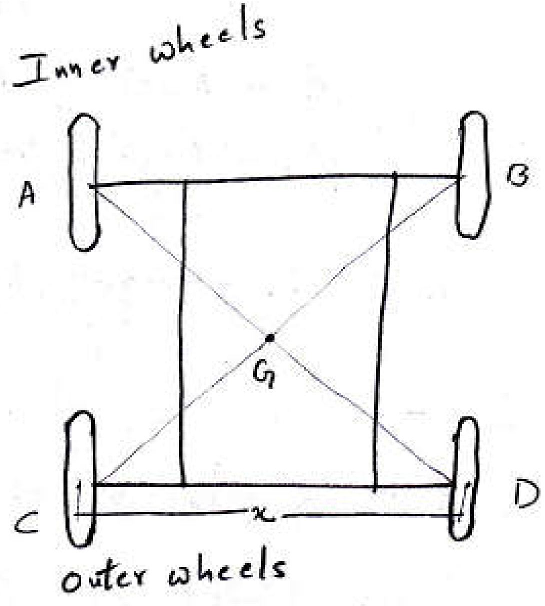
Force 1: weight of the automobile

- Let "W" be the weight of the automobile
- The weight of the automobile always acts in downward direction, vertically downwards.
- Magnitude of reaction force..?
  - Assuming all the wheels share equal load of the automobile and hence the reaction at each wheel is $\frac{W}{4}$
- Direction of reaction force..?
  - Vertically upwards.

Force 2: Gyroscopic force

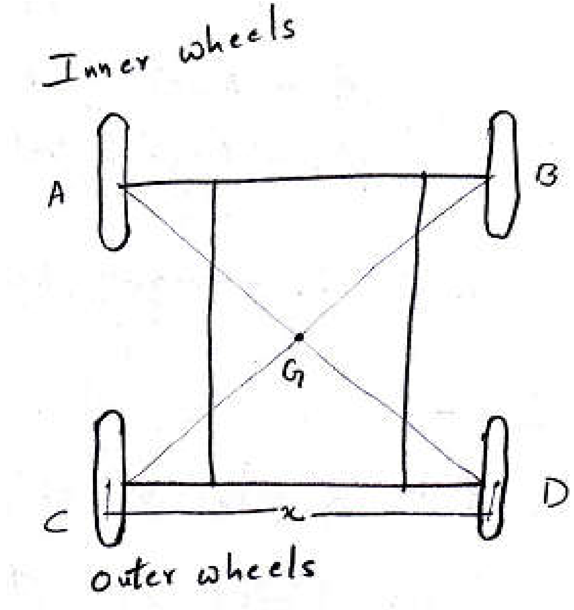
- In our earlier discussion we had derived the expression for the magnitude of gyroscopic couple which is $C_G = I.\ \omega\ \omega_p$
- But this is couple, what we require is force.
- We know that couple = force × perpendicular distance.
- The gyroscopic couple will be created in the system when we have two simultaneous rotational motions
- So in our automobile, what are the rotating components..?
  - Wheels
  - Engine rotating components.
- Hence the gyroscopic couple will be created by those components of the system.
  - C_W and C_E

Force 2: Gyroscopic force



- The magnitude of gyroscopic couple created by wheels and engine components is
  - $C_W = I_w \omega_w \omega_p \times$ no of wheels
  - $C_E = I_E \omega_E \omega_p$
- Direction analysis:
  - Let X axis be the axis of spin of the wheels
  - If the automobile take a turn in either left/right, Y will be axis of precession
  - Hence Z axis represents the axis of gyroscopic couple.
- Therefore at points A, B, C, D the gyroscopic couple acts in vertical direction trying to rotate the system.
- The total gyroscopic force = C = $C_E \pm C_w$

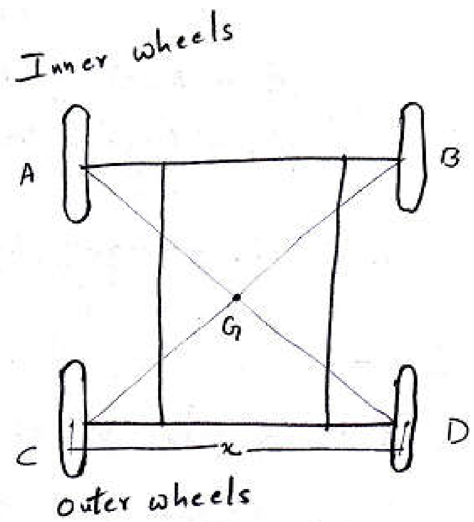
Force 2: Gyroscopic force



- So total gyroscopic couple is
  - $C = 4\, I_w\, \omega_w\, \omega_p \pm I_E\, \omega_E\, \omega_p = \omega_w\, \omega_p\, [4\, I_w \pm G.I_E]$
- Let 'P' be the force acting on both the inner/outer wheels
- Couple = force (P) × perpendicular distance (x)
- Hence gyroscopic force $P = \dfrac{Total\ couple\ (C)}{perpendicular\ distance\ (x)}$
- So now we know the magnitude of this gyroscopic force, now we need to find the direction
- Direction:
  - Due to this gyroscopic couple occurred while vehicle is turning, the whole system will try to overturn about Z axis
  - The gyroscopic couple will apply vertically downward force (P) on inner wheels and upward force on outer wheels (P) creating a couple action.

Force 2: Gyroscopic force

- Hence the reaction force due to this gyroscopic couple force on the automobile wheels is
  - Downward force on inner wheels
  - Upward force on outer wheels.
- The magnitude of gyroscopic force on each individual wheels is $\frac{P}{2}$
- So at points A, C which are inner wheels the gyroscopic force will act in downward direction and at points B, D it acts in upward direction.



Force 3: centrifugal force

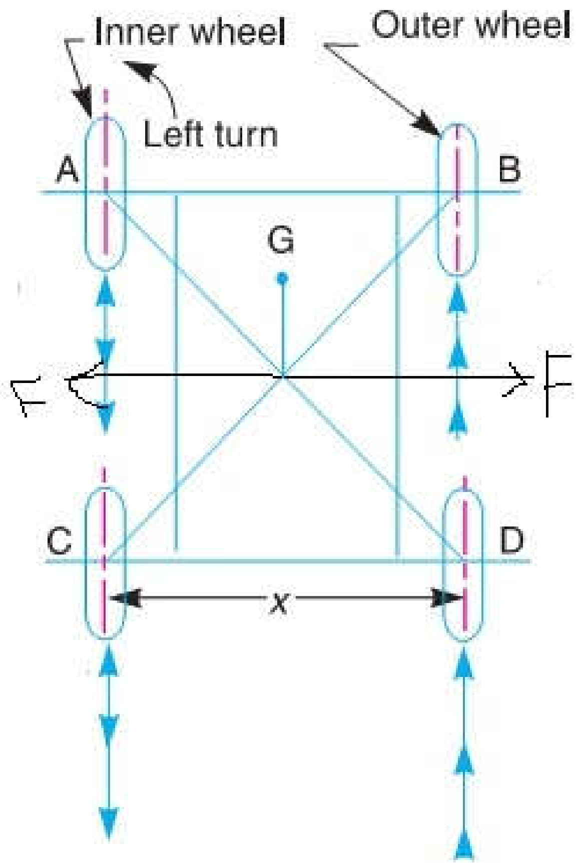
- All rotating systems will experience centrifugal force in radially outward direction.
- Since the automobile is trying to turn in a curved path, centrifugal force acts on the system.
- Again as the force is vector, we need to find its magnitude and direction.
- The magnitude of centrifugal force acting on the system is $F_C = \frac{m.V^2}{R}$
- This magnitude of centrifugal force acts from the center position of the automobile.
- But we are interested to find the magnitude of the force acting at the wheels… so how to find that..?

Force 3: centrifugal force

- To answer our question, we need to clear about another small concept…
- What kind of force is acting on the wheels in order to make it move in a curved path…?
  - Couple
- So let us find the magnitude of the couple created by the centrifugal force.
- Couple due to centrifugal force = $F_c \times$ perpendicular distance
- Now we have another question here … what is perpendicular distance ..?
- So in order to know about the perpendicular distance, we need to know in which axis-direction F_C acts.

Force 3: centrifugal force

- As the vehicle is turning in right/left, the centrifugal force acts in those direction.
- let "h" be the distance of centroid of the automobile from ground level.
- So the perpendicular distance = h
- So the couple due to centrifugal force = $F_c \times h$
- Now this magnitude of centrifugal couple is acting on the four wheels.
- We need to convert this centrifugal couple in to a force which acts on the wheels in Z axis direction.
- Let Q be that force acting on 2 inner/outer wheels.



Force 3: centrifugal force

- So Q = $\frac{centrifugal\ couple}{perpendicular\ distance}$
- So this perpendicular distance = x
- Hence Q = $\frac{F_c \times h}{x}$
- So now we had obtained the magnitude of force due to centrifugal couple acting at the wheels.
- As force is vector, we need to find its direction also.
- As discussed for gyroscopic force, the direction of force due to centrifugal couple (Q) also acts downwards for inner wheels and upwards for outer wheels.
- Hence each wheel experience $\frac{Q}{2}$ magnitude of force and acts downwards for inner wheel and upward for outer wheels.

Resultant forces on wheels

- Force is a vector, so it has both direction & magnitude.
- We discussed that forces ad their respective magnitudes acting on the system.
- The forces and their magnitudes are:
  - Weight acting on each wheels $\left(\frac{W}{4}\right)$ in upward direction.
  - Gyroscopic force on all the wheels having magnitude $\left(\frac{P}{2}\right)$
    - For inner wheels, in downward direction.
    - For outer wheels, in upward direction.
  - Centrifugal force on all wheels having magnitude $\left(\frac{Q}{2}\right)$
    - For inner wheels, in downward direction.
    - For outer wheels, in upward direction.

Analysis of Four Wheel automobile

- Total Resultant forces on Wheels
- Total force at inner Wheels = $\frac{W}{4} - \frac{P}{2} - \frac{Q}{2}$
- Total force at outer wheels = $\frac{W}{4} + \frac{P}{2} + \frac{Q}{2}$

Automobile taking a left turn

Resultant force for Inner wheels	Weight acting upwards	Gyroscopic force acting downwards	Centrifugal force acting downwards
Resultant force for Outer wheels	Weight acting upwards	Gyroscopic force acting upwards	Centrifugal force acting upwards

Analysis of a Two Wheel automobile

- Aim:
  - To maintain stability of the automobile.
  - To find the balancing mass.
- When balancing is required in case of two wheel automobiles?
  - While moving in straight path (no gyroscopic force acts)
  - While taking a turn (when gyroscopic force acts on the system)
- When moving in straight path how balancing of vehicle done?
  - Just sit straight on the bike and done move unnecessarily.
- While taking a turn how to do balancing ?
  - During turning as per our discussion gyroscopic force acts on system.
  - So the person should tilt in the opposite direction making him as a counter weight creating a balancing couple to make the system stable.